

# OPTIMISATION OF A MULTI-PRODUCT GREEN SUPPLY CHAIN MODEL WITH HARMONY SEARCH

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**Abstract:** *This study develops a methodological approach for optimisation of processes of green supply chain. It first looks into the processes of green supply chain and defines the most important functional entities, such as components suppliers, manufacturer, wholesaler, retailer, end user, collection system, recycling and disassembly plant, service stations and disposal plants. It proposes a mathematical description of the multi-product model, which includes the following financial oriented parameters: revenue, warehousing cost, transportation cost, recycling cost, subsidy, disposal cost and prices. In the second part of the paper, an optimisation method for the solution of the proposed hard NP problem is also discussed. This method is based on the harmony search algorithm, which imitates the musical performance process which takes place when a musician searches for a better harmony. The result of this study can be used to improve the utilisation of existing green supply chain from the point of view cost efficiency.*

**Key words:** *green supply chain, logistics, optimisation*



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## **1. Introduction**

Today the production and service processes of the economy are undergoing significant changes. Besides globalisation of these processes the increased importance of environmental management systems, recycling and clean technology can be witnessed. These changes have recently led to the increase of complexity of the technological and logistic processes, which means significant problems and increased costs for manufacturers, service enterprises (logistic centres, freight forwarding companies, etc.). The appearance of the environmental management systems led to the greening of supply chains. The optimal design of the green supply chain is a knowledge intensive task because of the complexity of the system. The huge number of entities and the connections of them define the problem field, and there are several aspects to find the optimal solution. Within the frame of this work, the author focuses on the optimisation of the whole green supply chain from the point of view logistics and logistics oriented costs. The variegation of supply chain concepts has to meet not only requirements of technology but also of industrial, trade and service fatigue, logistics, while creates the green supply chain to supply the needs of the market and the requirements defined by environmental law.

## **2. Problem definition**

Supply chain processes include the functions of procurement, collection, production, sales, distribution and recycling. The success of the operation of these functions is based on the optimal design and control. Most of the supply chain models include only the traditional procurement-production-distribution sub-processes used for converting raw materials to final products and deliver them to the wholesalers, retailers or directly to end users. The purpose of this paper is to show a possible complex green supply chain model which integrates the traditional supply chain and the reverse supply chain including collection, disassembly, recycling, second material market and disposal functions. The complexity of the purposed model is so complex that sophisticated operation research heuristics has to be used to find the optimal solution of the optimisation problem, which is the minimisation of the costs. There are several heuristics algorithms to solve NP and hard NP problems. In this case as a suitable method the harmony search algorithm (HSA) was chosen.

## **3. Literature review**

Much literature has been published in the field of green supply chain (GSC) and some of them have included a comprehensive survey (Sarkis et al., 2011). The previous studies of optimisation of green supply chain can be categorized into two main streams. The first stream addressed the development of traditional purchasing – production – distribution logistics chain models into closed loop economy (purchasing – production – distribution – recycling) models. Some generalised terms were created and published in different research works: sustainable supply network management (Cruz & Matsypura, 2009); supply and demand sustainability (Kovacs,

2004); supply chain environmental management (Sharfman et al., 2009); green purchasing (Min and Galle, 1997), environmental purchasing (Zsidisin & Siferd, 2001), green logistics (Murphy & Poist, 2000), and sustainable supply chains (Linton et al., 2007), JIT supply with virtual enterprises (Bányai, 2009). The IT aspects of networking entities are very important and there are some research works summarising aspects that have to be considered during design and set-up of IT structure of large scale networks, such as green supply chain: aspects of connections through IP-based networks in industrial environments (Koenig & Seminsky, 2009), possibilities of information logistics utilisation (Kiss et al., 2010). The economical growth can influence the efficiency, functionality and transparency of large systems based on a huge number of entities (end users), and therefore it is important to research the effects of the economical growth (Juratoni & Bundau, 2009). The green supply chain has very high costs and one of the most important costs of these large scaled systems is the transportation cost. Therefore it is important to optimise the transportation processes of these systems from the point of view transportation technology and logistics (Mlinaric et al., 2008).

The second stream addressed the development of optimisation methods from the point of view increasing of efficiency of closed loop economy. The reduced lifecycle due to both technological advancements and environmental concerns led to the development of system dynamics model for dynamic capacity planning of remanufacturing in closed-loop supply chains (Vlachos et al., 2007 and Pupavac & Sehanovic, 2007). The analysis of a generic system was realised by the aid of a simulation based model with application of system dynamics methodology, which is an excellent tools to study problems that arise in closed-loop systems. However the development of this generic system takes into consideration not only the economic, but also environmental issues, but the focus is the single product supply chain, which does not include secondary material market. In a global economy, one of the key factors of successfully design and operation is taking into consideration the effects of uncertainty. The optimisation of a closed loop supply chain at more planning periods, while accounting for the importance of each partner in the global chain as well as demand/price uncertainties is possible by the aid of mixed integer linear programming using a standard branch and bound procedure (Amaro & Barbosa-Povoa, 2009). There are some green supply chain models in the literature, but most of them focus on single product systems and the models are basically linear multi-objective programming models (Sheu et al, 2009). There are a huge number of research works, which focus on the optimisation of a special application area of the optimisation of closed loop economy: nuclear power generation by the aid of linear multi-objective optimisation (Sheu, 2008), environmental-regulation pricing strategies by the aid of analytic solutions of Markovian Nash equilibriums derived from Hamilton–Jacobi–Bellman equations (Chen & Sheu, 2009), integrated eco-management in service areas, especially in hotels (Sztruten et al., 2010). Important aspect of the topic is the integration of knowledge into the education (Bányai, 2009a).

To the authors knowledge there has been no research work on multi product green supply chain that take into consideration the logistics parameters of the whole system including a huge number of entities (component suppliers, manufacturers,

wholesalers, retailers, end users, collection system, recycling and disassembly plant, service stations, disposal plant) of the GSC. Authors describe a mathematical model and the optimisation of multi product green supply chain from the point of view optimisation of profit taking into consideration logistic parameters.

#### 4. Model of green supply chain

The model of the green supply chain includes all entities of the functional logistic entities (purchasing, production, distribution and recycling) of closed loop economy: the supplier network (sources of components required for products), manufacturers, commercial subsystem of the supply chain including wholesalers and retailers, end users, service stations, collection system, disassembly and recycling plants, secondary material market and disposal plants.

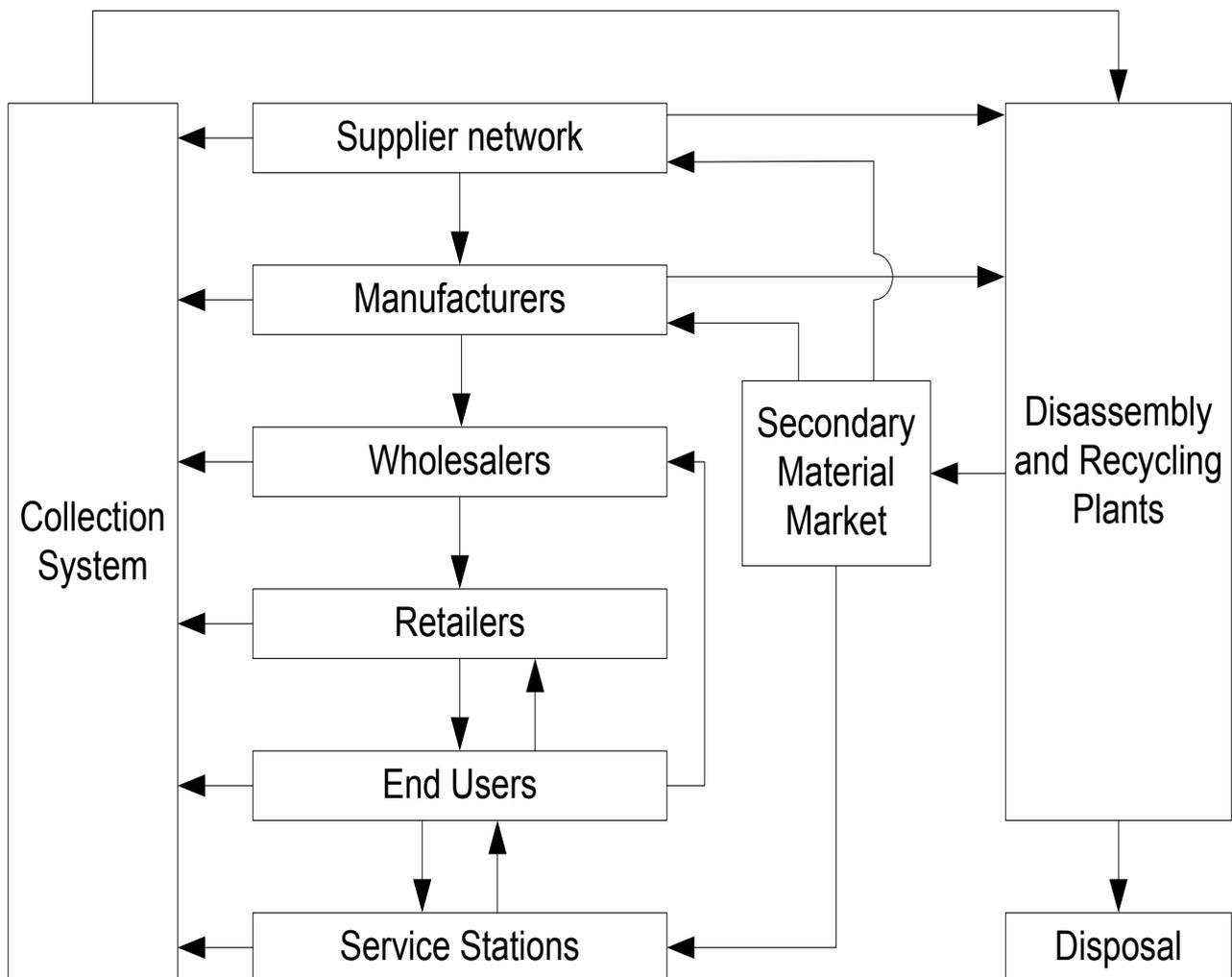


Fig. 1. Model of green supply chain

During the optimisation of the above mentioned green supply chain model (see Fig. 1.) the following assumptions have to be taken into consideration: (a) In the proposed model the multi-product condition is considered. (b) The manufacturer produce products from components purchased from either the component suppliers or

the secondary material market. The proportion of new, used and refurbished products is given. (c) The manufacturer transports the products from the output warehouse directly to the wholesalers. (d) The refuse ratio of products produced by the manufacturer depends on the quality of the purchased components; the refurbished components have higher refuse rate than new components. (e) The return ratio of used products from end users to wholesalers, retailers, services and entities of collection system is given. (f) The direct return of used products from end users to recycling and disassembly plants is not allowed. (g) The service stations, component suppliers and manufacturers can purchase used and/or refurbished components from the secondary material market. (h) The specific warehousing cost depends on the type of the components (new, used or refurbished). The specific warehousing cost of used products is lower in the case of new components, because of the special warehousing conditions in the case of used products (e.g. wastes of electric and electronic components).

The revenue of the manufacturer depends on the sold amount of products ( $Q_i^{L1}$ ) and the lot size of transportation ( $q_i^{L1}$ ):

$$R^{L1} = \sum_{i \in \Psi} Q_i^{L1} * \left( R_i^{L1,min} + (R_i^{L1,max} - R_i^{L1,min}) * \left( \frac{q_i^{L1,UB} - q_i^{L1}}{q_i^{L1,UB} - q_i^{L1,LB}} \right) \right) \quad (1)$$

The warehousing cost of the manufacturer has two important parts: the warehousing costs of new components purchased from the components supplier ( $WC^{L1}$ ) and the warehousing cost of refurbished components purchased from the second material market ( $WC_2^{L1}$ ):

$$WC^{L1} = WC_1^{L1} + WC_2^{L1} \quad (2)$$

The warehousing cost at the manufacturers depends on the warehousing cost of final products and the warehousing cost of purchased components. The first term on the left handside of the Eq.(3) represents the warehousing cost of purchased components, which depends on the lot size of transportation of new components from component suppliers to manufacturers ( $q_j^{L1,p}$ ). The right handside of the same equation represents the warehousing cost of final products, which depends on the proportion of the number of purchased components from source p and the total number of components ( $\alpha_{i,j}^p$ ), the structure of the products and the unit warehousing cost of components from suppliers ( $\kappa_j^{L1,W,p}$ ):

$$WC_1^{L1} = \sum_{j \in \Theta_i} \sum_{p \in \Phi} \left| \frac{q_j^{L1,p} * \tau}{2} - \sum_{i \in \Psi} \frac{\alpha_{i,j}^p * n_{i,j} * q_i^{L1}}{2} * \tau \right| * \kappa_j^{L1,W,p} \quad (3)$$

The warehousing cost of refurbished components depends on the sold amount of final products, the refuse rate of components ( $\eta_{i,j}^{L1,p}$ ), the amount of refused products transported to the collection system, the intensity of the transportation of refused products from the manufacturer to the collection system or to the recycling and disassembly plant and the unit warehousing cost of refused products ( $\kappa_i^{L1,W,r}$ ):

$$WC_2^{L1} = \sum_{i \in \Psi} Q_i^{L1} * \left( \prod_{j \in \Theta_i} (1 + \eta_{i,j}^{L1,p})^{n_{i,j}} - 1 \right) * \frac{\tau}{2} * \left( \frac{\beta_1}{z_1} + \frac{1 - \beta_1}{z_2} \right) * \kappa_i^{L1,W,r} \quad (4)$$

The transportation cost of components from the component sources to the manufacturer consists of the cost of three different transportation relations: (1) the transportation cost of components from component sources to manufacturers; (2) the transportation cost of refused components and products from manufacturers to collection system and (3) the transportation cost of refused components and products from manufacturers to disassembly plant:

$$TC^{L1} = TC_1^{L1} + TC_2^{L1} + TC_3^{L1} \quad (5)$$

The transportation cost of components from component sources to manufacturers depends on the lot size of transportation of new components from component suppliers to manufacturers ( $q_j^{L1,p}$ ), the maximum load capacity of the truck from component source p to manufacturer ( $c_{truck}^{L1,p}$ ), the specific transportation cost of components in the case of one truck from supplier p defined by set  $\Phi$  (set  $\Phi$  defines the set of different new component suppliers of the green supply chain), the total amount of components purchased by the manufacturer from component sources ( $Q_j^{L1,p}$ ). This equation makes it possible to take into consideration the periodicity of the transportation cost depending on the number of trucks.

$$TC_1^{L1} = \sum_{j \in \Theta_i} \sum_{p \in \Phi} \left( \frac{q_j^{L1,p}}{c_{truck}^{L1,p}} \right)_{int} * \frac{Q_j^{L1,p}}{q_j^{L1,p}} * \kappa_j^{L1,T,p} \quad (6)$$

The transportation cost of refused components and products from manufacturers to collection system depends on the sold amount of final products, the refuse rate of components ( $\eta_{i,j}^{L1,p}$ ), the amount of refused products transported to the collection system, the intensity of the transportation of refused products from the manufacturer to the collection system and the unit transportation cost of refused products ( $\kappa_i^{L1,T,r}$ ):

$$TC_2^{L1} = \left( \sum_{i \in \Psi} Q_i^{L1} * \left( \prod_{j \in \Theta_i} (1 + \eta_{i,j}^{L1,p})^{n_{i,j}} - 1 \right) * \frac{\beta_1}{z_1 * c_{truck}^{L1,r}} \right)_{int} * z_1 * \kappa_{i,1}^{L1,T,r} \quad (7)$$

The transportation cost of refused components and products from manufacturers to disassembly plant depends on the same parameters as in the case of the transportation cost of refused components and products from manufacturers to collection system, but in this case the manufacturer- recycling and disassembly plant relation has to be taken into consideration:

$$TC_3^{L1} = \left( \sum_{i \in \Psi} Q_i^{L1} * \left( \prod_{j \in \Theta_i} (1 + \eta_{i,j}^{L1,p})^{n_{i,j}} - 1 \right) * \frac{1 - \beta_1}{z_2 * c_{truck}^{L1,r}} \right)_{int} * z_2 * \kappa_{i,2}^{L1,T,r} \quad (8)$$

The manufacturing process has a refuse rate depending on the refuse rate and number of built-in components. However the physical process of the recycling of refused final products means the transportation of refused products from the manufacturer to the collection system or to the recycling and disassembly plant, but the cost of this transportation belongs to the transportation cost. The recycling cost is the recycling fee paid by the manufacturer as a product fee, which depends on the on the refuse rate of components and the unit recycling cost of different products ( $rc_i^{L1}$ ):

$$RC^{L1} = \sum_{i \in \Psi} Q_i^{L1} * rc_i^{L1} * \left( \prod_{j \in \Theta_i} (1 + \eta_{i,j}^{L1,p})^{n_{i,j}} - 1 \right) \quad (9)$$

The production cost of manufacturers includes the aggregate assembly cost of components and depends on the number of build-in components, the sold amount of products ( $Q_i^{L1}$ ) and the refuse rate of components ( $\eta_{i,j}^{L1,p}$ ):

$$PC^{L1} = \sum_{i \in \Psi} \left( Q_i^{L1} * \prod_{j \in \Theta_i} (1 + \eta_{i,j}^{L1,p})^{n_{i,j}} * \sum_{p \in \Phi} \sum_{j \in \Theta_i} n_{i,j} * \varepsilon_{i,j}^{L1,p} \right) \quad (10)$$

The price of components to be paid by the manufacturer to the component supplier and second material market depends on ratio of new and refurbished components ( $\zeta_j$ ), the amount and specific costs:

$$PR^{L1} = \sum_{p \in \Phi} \sum_{j \in \Theta_i} \zeta_j * q_j^{L1,p} * pr_j^{L1,p,n} + (1 - \zeta_j) * q_j^{L1,p} * pr_j^{L1,r} \quad (11)$$

From the above mentioned costs (warehousing, transportation, production or assembly, recycling and purchasing), the profit of the manufacturer is generated:

$$P^{man} = R^{L1} - WC^{L1} - TC^{L1} - PC^{L1} - RC^{L1} - PR^{L1} \quad (12)$$

The revenue of the wholesalers depends on the sold amount of products to the retailers ( $Q_i^{L2}$ ) and the lot size of transportation ( $q_i^{L2}$ ):

$$R^{L2} = \sum_{i \in \Psi} Q_i^{L2} * \left( R_i^{L2,min} + (R_i^{L2,max} - R_i^{L2,min}) * \left( \frac{q_i^{L2,UB} - q_i^{L2}}{q_i^{L2,UB} - q_i^{L2,LB}} \right) \right) \quad (13)$$

The warehousing cost of the wholesaler has two important parts: the warehousing costs of new products purchased from the manufacturer and the warehousing cost of used products from end users. The first term on the left handside of the Eq.(14) represents the warehousing cost of new products, which depends on the lot size of transportation from manufacturer to the wholesaler ( $q_i^{L1}$ ) and the sold lot from the wholesaler to the retailer ( $q_i^{L2}$ ) and the unit warehousing cost of used products at the wholesaler ( $\kappa_i^{L2,W}$ ). The right handside of the same equation represents the warehousing cost of used, recollected products from end users, which depends on the different transportation lots of used product in the relations end user-wholesaler and wholesaler-collection system, and the specific warehousing cost of used products ( $\kappa_i^{L2,W,r}$ ).

$$WC^{L2} = \sum_{i \in \Psi} \left( \frac{|q_i^{L1} - q_i^{L2}|}{2} * \kappa_i^{L2,W} + \frac{|q_i^{L1,r} - q_i^{L2,r}|}{2} * \kappa_i^{L2,W,r} \right) * \tau \quad (14)$$

The wholesaler has two important transportation processes: the transportation of new products to the retailers and the transportation of used products to the collection system. The left handside of the Eq.(15) represents the transportation cost of new products to the retailers, which depends on the transportation lot ( $q_i^{L2}$ ), the maximum load capacity of the truck from wholesaler to the retailer ( $c_{truck}^{L2}$ ), the specific transportation cost ( $\kappa_i^{L2,T}$ ), and the total amount of components sold to the retailer ( $Q_i^{L2}$ ). The right handside of the same equation describe the transportation cost of used, recollected products to the collection system, which depends on the transportation lot ( $q_i^{L2,r}$ ), the maximum load capacity of the truck from wholesaler to the retailer ( $c_{truck}^{L2,r}$ ), the specific transportation cost ( $\kappa_i^{L2,T,r}$ ), and the total amount of recollected used products ( $Q_i^{L2,r}$ ).

$$TC^{L2} = \sum_{i \in \Psi} \left( \left( \frac{q_i^{L2}}{c_{truck}^{L2}} \right)_{int} * \frac{Q_i^{L2}}{q_i^{L2}} * \kappa_i^{L2,T} + \left( \frac{q_i^{L2,r}}{c_{truck}^{L2,r}} \right)_{int} * \frac{Q_i^{L2,r}}{q_i^{L2,r}} * \kappa_i^{L2,T,r} \right) \quad (15)$$

The recycling cost of the wholesaler depends on the quantity of recollected used products and the specific recycling cost ( $rc_i^{L2}$ ):

$$RC^{L2} = \sum_{i \in \Psi} Q_i^{L2,r} * rc_i^{L2} \quad (16)$$

The price of the products at the wholesaler is the function of the revenue of the manufacturer with respect to the taxes and other additional appurtenances:

$$PR^{L2} = R^{L1} * \varpi_1 \quad (17)$$

From the above mentioned costs (warehousing, transportation, recycling and purchasing), the profit of the wholesaler is generated:

$$P^{who} = R^{L2} - WC^{L2} - TC^{L2} - RC^{L2} - PR^{L2} \quad (18)$$

The revenue of the retailer depends on the sold amount of products to the end users ( $Q_i^{L3}$ ) and the lot size of transportation ( $q_i^{L3}$ ):

$$R^{L3} = \sum_{i \in \Psi} Q_i^{L3} * \left( R_i^{L3,min} + (R_i^{L3,max} - R_i^{L3,min}) * \left( \frac{q_i^{L3,UB} - q_i^{L3}}{q_i^{L3,UB} - q_i^{L3,LB}} \right) \right) \quad (19)$$

The warehousing cost of the retailer has two important parts: the warehousing costs of new products purchased from the wholesaler and the warehousing cost of used products from end users. The first term of the Eq.(20) represents the warehousing cost of new products, which depends on the transportation lot from wholesaler to the retailer ( $q_i^{L3}$ ) and the sold lot from the retailer to the end user ( $q_i^{L3}$ ) and the unit warehousing cost of used products at the wholesaler ( $\kappa_i^{L3,W}$ ). The right handside of the same equation represents the warehousing cost of used, recollected products from end users, which depends on the different transportation lots of used product in the relations end user-retailer and retailer-collection system, and the specific warehousing cost of used products ( $\kappa_i^{L3,W,r}$ ).

$$WC^{L3} = \sum_{i \in \Psi} \left( \frac{|q_i^{L2} - q_i^{L3}|}{2} * \kappa_i^{L3,W} + \frac{|q_i^{L2,r} - q_i^{L3,r}|}{2} * \kappa_i^{L3,W,r} \right) * \tau \quad (20)$$

There are two main transportation activities at the retailer: the first one is the transportation of new products to the end users and the second one is the transportation of used products to the collection system. The left handside of the Eq.(21) represents the transportation cost of new products to the end users, which depends on the transportation lot ( $q_i^{L3}$ ), the maximum load capacity of the truck from wholesaler to the retailer ( $c_{truck}^{L3}$ ), the specific transportation cost ( $\kappa_i^{L3,T}$ ), and the total amount of components sold to the retailer ( $Q_i^{L3}$ ). The right handside of the same equation describe the transportation cost of used, recollected products to the collection system, which depends on the transportation lot ( $q_i^{L3,r}$ ), the maximum load capacity of the truck ( $c_{truck}^{L3,r}$ ), the specific transportation cost ( $\kappa_i^{L3,T,r}$ ).

$$TC^{L3} = \sum_{i \in \Psi} \left( \left( \frac{q_i^{L3}}{c_{truck}^{L3}} \right)_{int} * \frac{Q_i^{L3}}{q_i^{L3}} * \kappa_i^{L3,T} + \left( \frac{q_i^{L3,r}}{c_{truck}^{L3,r}} \right)_{int} * \frac{Q_i^{L3,r}}{q_i^{L3,r}} * \kappa_i^{L3,T,r} \right) \quad (21)$$

The recycling cost of the retailer depends on the quantity of recollected used products and the specific recycling cost ( $rc_i^{L3}$ ):

$$RC^{L3} = \sum_{i \in \Psi} Q_i^{L3,r} * rc_i^{L3} \quad (22)$$

The price of the products at the retailer is the function of the revenue of the manufacturer with respect to the taxes and other additional appurtenances:

$$PR^{L3} = R^{L2} * \varpi_2 \quad (23)$$

From the above mentioned costs (warehousing, transportation, recycling and purchasing), the profit of the retailer is generated:

$$P^{ret} = R^{L3} - WC^{L3} - TC^{L3} - RC^{L3} - PR^{L3} \quad (24)$$

To calculate the recycling costs of collection system the first step is to describe the collected and recycled amount of used products from end users to different collection entities. End users can transport used products four different parts of the green supply chain: wholesalers, retailers, collection system and service stations. Eq.(25-27) describe the quantities of these amounts.

$$Q^{rec} = Q^{rec,w} - Q^{rec,ret} - Q^{rec,cs} - Q^{re,ser} \quad (25)$$

$$Q_i^{rec,w} = Q_i^{L3} * \xi_i^w, \quad Q_i^{rec,ret} = Q_i^{L3} * \xi_i^{ret} \quad (26)$$

$$Q_i^{rec,cs} = Q_i^{L3} * \xi_i^{cs}, \quad Q_i^{rec,ser} = Q_i^{L3} * \xi_i^{ser} \quad (27)$$

The collection system receives used products from the following entities of the green supply chain: supplier network, manufacturer, wholesaler, retailer, end user, service stations. The revenue of the collection system is the subsidy:

$$R^{L4} = \sum_{i \in \Psi} S_i^{L4} \quad (28)$$

The warehousing cost of the collection system depends on the frequency (or lot) of transportation of used products from supplier network, manufacturer, wholesaler, retailer, end users and service stations ( $q_i^m$ ), the transportation lot of collected products to the recycling and disassembly plant ( $q_i^{L4}$ ) and the specific warehousing cost of used products at the collection system ( $\kappa_i^{L4}$ ).

$$WC^{L4} = \sum_{i \in \Psi} \sum_{m \in \Omega} \frac{q_i^m - q_i^{L4}}{2} * \tau * \kappa_i^{L4} \quad (29)$$

The transportation cost of the collection system includes the cost of the transportation of used products from the retailer to the disassembly and recycling plant, and depends on the transportation lot of collected products to the recycling and disassembly plant, the maximum load of the truck from the collection system to the disassembly and recycling plant and the specific transportation cost ( $\kappa_i^{L4,T}$ ).

$$TC^{L4} = \sum_{i \in \Psi} \left( \frac{q_i^{L4}}{C_{truck}^{L4}} \right)_{int} * \frac{Q_i^{L4}}{q_i^{L4}} * \kappa_i^{L4,T} \quad (30)$$

From the above mentioned warehousing and transportation costs and the revenue based on subsidy the profit of the retailer is generated as follows:

$$P^{col} = R^{L4} - WC^{L4} - TC^{L4} \quad (31)$$

The revenue of the disassembly and recycling plant can be described on the basis of Eq. (32). The left handside of the equation represents the subsidy and the right part is the revenue from the sale of components, materials and products on the secondary material market.

$$R^{L5} = \sum_{i \in \Psi} (S_i^{L5} + R_i^{SMM}) \quad (32)$$

The warehousing cost of disassembly and recycling plant is a function of the frequency of transportation of used products from collection system to recycling and disassembly plant, from recycling and disassembly plant to disposal plant ( $q_i^{L6}$ ) and secondary material market ( $q_i^{L5}$ ). The specific warehousing costs ( $\kappa_i^{L6}$ ) are the same in each solution.

$$WC^{L5} = \sum_{i \in \Psi} \frac{q_i^{L4} - q_i^{L5} - q_i^{L6}}{2} * \tau * \kappa_i^{L6} \quad (33)$$

There are two main transportation activities at the recycling and disassembly plant: the first one is the transportation of disassembled not recyclable components or materials of used products to the disposal plant and the second one is the transportation of refurbished products or components to the collection system. The left handside of the Eq.(34) represents the transportation cost of refurbished products or components to the secondary material market, which depends on the transportation lot ( $q_i^{L5}$ ), the maximum load capacity of the truck ( $c_{truck}^{L5}$ ), the specific transportation cost ( $\kappa_i^{L5,T}$ ), and the total amount of refurbished products or components sold to the secondary material market ( $Q_i^{L5}$ ). The right handside of the same equation describe the transportation cost of not recyclable components or materials of used products to the disposal plant, which depends on the transportation lot ( $q_i^{L6,r}$ ), the maximum load capacity of the truck ( $c_{truck}^{L6}$ ), the specific transportation cost ( $\kappa_i^{L6,T}$ ), and the total amount of sold refurbished components or materials ( $Q_i^{L6}$ ).

$$TC^{L5} = \sum_{i \in \Psi} \left( \frac{q_i^{L5}}{c_{truck}^{L5}} \right)_{int} * \frac{Q_i^{L5}}{q_i^{L5}} * \kappa_i^{L5,T} + \left( \frac{q_i^{L6}}{c_{truck}^{L6}} \right)_{int} * \frac{Q_i^{L6}}{q_i^{L6}} * \kappa_i^{L6,T} \quad (34)$$

From the above mentioned warehousing and transportation costs, the revenue based on subsidy and the incomes from sales to second material market the profit of the recycling and disassembly plant is generated as follows:

$$P^{rec} = R^{L5} + S^{L5} - TC^{L5} - WC^{L5} \quad (35)$$

The composite multi-objective function of this optimisation problem is the maximisation of the total profit of the proposed green supply chain model including manufacturer, wholesaler, retailer, end user, collection system, recycling and disassembly plants:

$$C = P^{man} * \varphi^{man} + P^{who} * \varphi^{who} + P^{ret} * \varphi^{ret} + P^{col} * \varphi^{col} + P^{rec} * \varphi^{rec} \rightarrow min. \quad (36)$$

## 5. Optimisation of the model with harmony search algorithm

Harmony search is a meta-heuristic algorithm, mimicking the improvisation process of music players. There are several parameters of the harmony search algorithm, which makes it more efficient and flexible, than other meta-heuristics: (a) there are fewer mathematical conditions; (b) the initial setting of decision variables is not necessary; (c) derivative information is not required; (d) the new solution vectors are created using the information on all of the existing possible solution vectors (Lee & Geem, 2005 and Pan et al, 2010).

The harmony search process includes five steps: (a) initialisation of the mathematical problem and the initial value of algorithm parameters; (b) fill in the harmony memory matrix with randomly generated solutions of the objective function of the optimisation problem; (c) improvisation of the new harmony memory matrix using values of all existing harmony vectors; (d) update of harmony memory matrix; (e) repeat the above mentioned steps until termination criteria (Kang & Geem, 2004).

However there are some research works to try to eliminate algorithm parameters of heuristic algorithms, such genetic algorithm, simulated annealing, tabu search, ant colony optimization, particle swarm optimization but within the frame of this research work the parameterised harmony search algorithm will be described and used to find the optimal solution of the problem of green supply chain (Geem & Sim, 2010).

Eq. (36) represents the objective function of the optimisation. The most important parameters of the algorithm are the harmony memory size (maximum number of solution vectors in the harmony memory matrix, HMS), harmony memory considering rate (HMCR), pitch adjusting rate (PAR) and of course the termination criteria. The aim of the harmony memory considering rate and the pitch adjusting rate is to improve the fitness of solutions in the harmony memory matrix. Harmony memory consideration rate is the rate of choosing a value from the vectors stored in the harmony memory matrix:  $x'_i \leftarrow x'_i \in \{x_i^1, x_i^2, \dots, x_i^{HMS}\}$  with probability HMCR and  $x'_i \leftarrow x'_i \in X_i$  with probability  $1-HMCR$ . Pitch correction is the process of correcting the intonation of an audio signal without affecting other aspects of its sound. Every component is examined to determine whether it should be pitch corrected or pitch adjusted: pitch adjusting is necessary with probability PAR. The value of pitch adjustment means the band distance (Mahdavi et al., 2007).

The solution vector of this green supply chain problem consists of the different types of lots, for example the procurement lot of new components from component suppliers to manufacturers, the procurement lot of used and refurbished components from secondary material market to manufacturers, the transportation lot in the level of distribution (manufacturing – wholesaler – retailer – end user supply chain), the transportation lot of collected used products from entities of collection system to disassembly and recycling plant, transportation lot of used and refurbished products from disassembly and recycling plant to secondary material market and the lot of transportation of not refurbishable products, components and materials from disassembly and recycling plant of disposal plant. The solution vector includes the rate of new and refurbished components to be used to manufacture products.

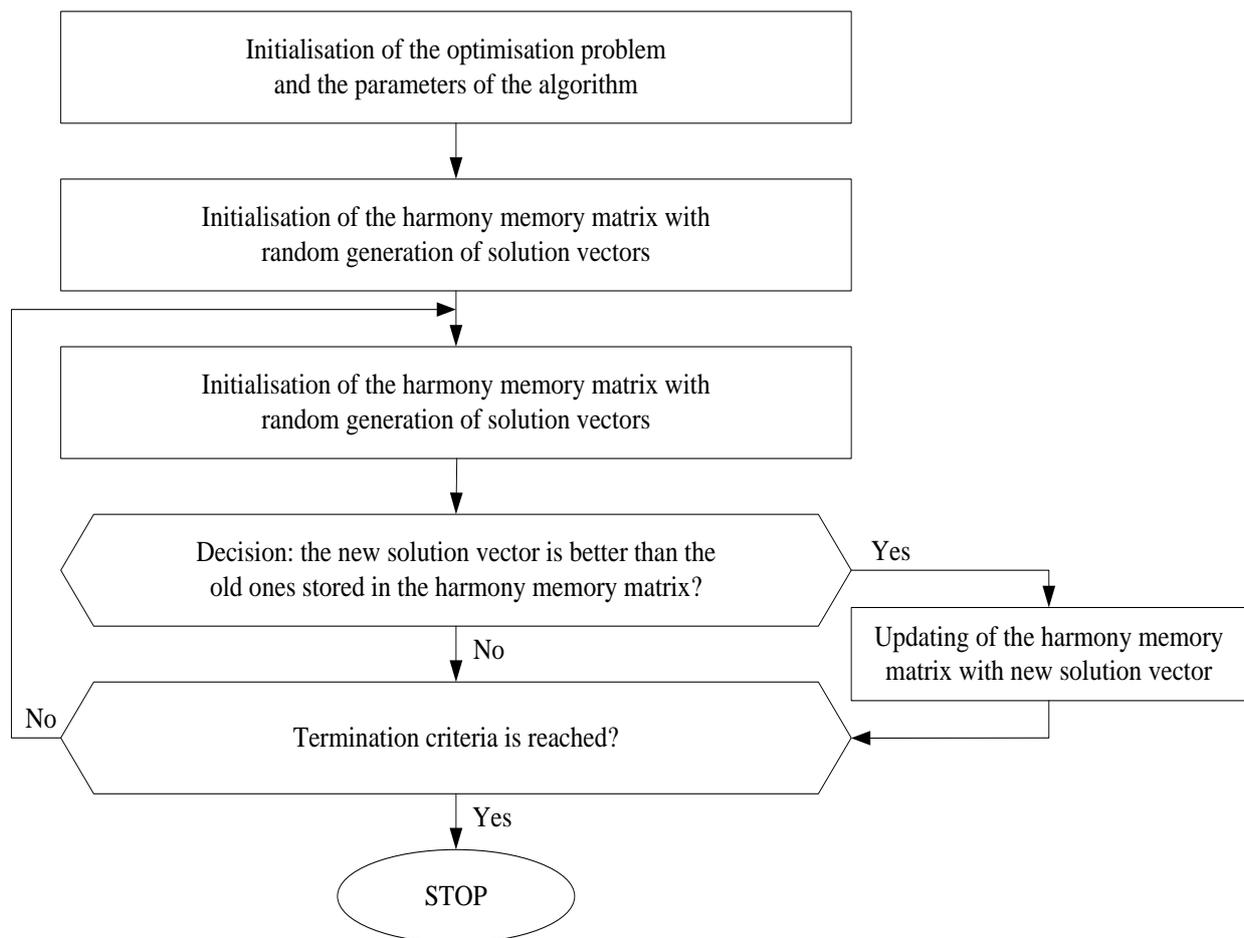


Fig. 2. Simplified optimisation procedure of the harmony search algorithm

By the aid of the above mentioned harmony search algorithm it is possible to solve the optimisation problem of the described complex green supply chain model. The algorithm is flexible and makes it possible to find a “good” solution vector, by the aid of which the optimal parameters of the logistic processes can be defined.

## 6. Conclusion, consequences and future research

The modelling and optimisation of supply chain is a very complex problem of the economy, but during the optimisation of these systems not only economical, but also logistic, aspects have to be taken into consideration. It is possible to build a complex model of supply chain including “green sub-processes” called green supply chain, but the mathematical model is so complex, that the solution is not possible by the aid of analytical methods. There are several heuristic algorithms, which make it possible to solve such complex models and find the optimal logistic and economical parameters of the operation. The above mentioned harmony search algorithm makes it possible to find a “good solution” of the optimisation problem.

In the case of implementation of the ideas presented in this article it would be possible to design more efficient green supply chain form the point of view logistics. The developed optimisation model and method would be more efficient in the case of integration into different enterprise systems. The future research direction is the development of the optimisation model to take into consideration more parameters of the real system.

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