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## Consideration of Influence of Peculiarities of Compressed Gas on Mathematical Model Parameters of Spool Valve

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### Abstract

In this paper is made an elaboration of the mathematical model of electro-pneumatic throttle spool-type valve, involving the theory of gas dynamics. The main idea of the elaboration is based on a consideration of peculiarities of gas flow through the valve, in terms of a gas flow behaving under an influence of geometry features of the spool-type valve. Emphasis was placed on a gas flow influence on spool dynamics, as well as on the consumption ratio of the valve. To check theoretical results was made a mathematical simulation of a gas flow in the package ANSYS Fluent.

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*Keywords:* electro-pneumatic valve; spool valve dynamics; consumption ratio; gas dynamics theory; mathematical model

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### 1. Introduction

Pneumatic machines and drives are widely used in different industrial areas but usually in open-loop systems. Invention of a pneumatic muscle has made possible use of pneumatic actuators in closed-loop automatic systems. As a result there were constructed few anthropomorphic manipulators which approximate statements of a human hand, for example, Airic's Arm (Festo) and Dexterous Hand (Shadow Robot Company). Nevertheless pneumatic muscles are almost not used in follow-up systems. One of possible reasons authors see in lack of reliable dynamic mathematical models of pneumatic muscles. But last time there were made some static mathematical models or empirical modelling of air muscle based actuators, e.g. [1–3].

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On the Automatic Control chair at Baltic State Technical University are made researches in creation of reliable dynamic mathematical models of different types of forced membrane elements [4, 5]. Although these models considerate gas influence on dynamic processes took place in elements, in them are used simplified mathematical models of valves taken from hydraulics. This approximation is correct only for air flows which velocity is less than 0.14 Ma. Furthermore even in hydraulics, where most systems are closed-loop, there are used by developers simplified models which not considerate fluid mechanics [6].

## 2. Proportional electro-pneumatic valve description

Pneumatic drive contains force part and regulator, which is in our case an electro-pneumatic spool valve proportional type. Valve itself consist of control solenoid and spool mechanism.

It is obvious that consideration of influence of peculiarities of compressed gas on mathematical model parameters of spool valve should be done. Generation of a such model of a valve makes possible creation of high-precision and fast-acting closed-loop pneumatic systems, e.g. vibration stands [7], active vibroprotection systems, anthropomorphic manipulators, etc. There were developed some mathematical models of proportional valves [8]. It is necessary to notice that such models are based on fitting valve parameters, that have correlation with gas flow parameters, to experimental values. In our case we are looking for creation of equations based on gas dynamics theory. As a result we are trying to develop universal valve mathematical model.

Transfer function of an electro-pneumatic spool valve one can write [9]:

$$W_{\text{ap}}(s) = \frac{Q(s)}{U(s)} = W_e(s) \cdot W_m(s) \cdot W_v(s) = \frac{k_{em}}{T_{em}s+1} \cdot \frac{k_m}{T_m^2s^2+\xi T_ms+1} \cdot k_v \quad (1)$$

where  $Q(s)$  – consumption;  $U(s)$  – control voltage;  $W_e(s)$  – transfer function of an electromagnetic part;  $W_m(s)$  – transfer function of a mechanical part;  $W_v(s)$  – transfer function of a valve (spool);  $k_{em}$  – transfer coefficient of electromagnetic part;  $k_m$  – transfer coefficient of mechanical part;  $k_v$  – transfer coefficient of volume flow rate;  $T_e$  – time constant of electromagnetic part;  $T_m$  – time constant of mechanical part;  $\xi$  – damping coefficient.

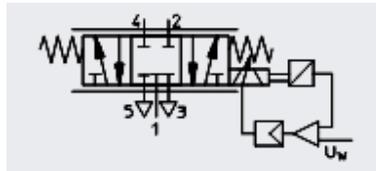


Fig. 1. Schematic circuit of the valve.

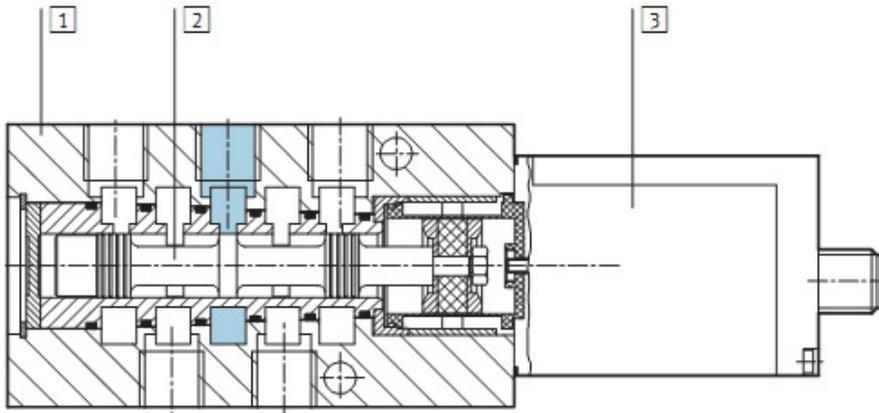


Fig. 2. Longitudinal section of the valve.

Let us see closer at influence of gas dynamics forces on valve dynamics. Gas-dynamics force is called because of changing of momentum of gas mass, which flows through spool cell, and jet-acting on a spool. Consideration of gas-dynamics forces will be carried out in two parts. In the first part we will consider the influence on transfer coefficient  $k_v$ , in the second – on parameters of transfer function of a mechanical part. Also there will be an assumption that processes in electromechanical part are so fast that could be neglected. If it is necessary to consider processes take place in electromagnetic part, this could be made using well-known electric theory. As an example was taken proportional spool valve Festo MYPE-5-1/8LF-010-B. Schematic circuit of the valve is in fig. 1, on fig. 2 – longitudinal section, where 1 – body, 2 – spool, 3 – electronic block.

### 3. Transfer function of the valve

In this part is considered an influence of gas-dynamic forces on transfer coefficient of volume flow rate, which one can write:

$$k_v = \frac{Q}{x_v}$$

Gas volume in pneumosystems strongly depend on pressure and temperature. For this reason it is more correct to obtain relations for mass flow rate, and then return to volume flow rate with following formula:

$$Q = \frac{G}{\rho} = \frac{G \cdot R \theta}{p}, \quad (2)$$

where  $G$  – mass flow rate;  $\rho$  – density,  $R$  – universal gas constant,  $\theta$  – absolute temperature,  $p$  – pressure. Mass flow rate through valve usually one can calculate [10]:

$$G = G^* \cdot f\left(\frac{p_b}{p_a}\right), \quad (3)$$

where  $G^*$  - critical mass flow rate ( $Ma \approx 1$ );  $p_a$  – constant input pressure;  $p_b$  – constant output pressure. For air [8] there are following approximate formulas:

$$f\left(\frac{p_b}{p_a}\right) = \begin{cases} 2 \sqrt{\left(1 - \frac{p_b}{p_a}\right) \frac{p_b}{p_a}}, & \text{when } \frac{p_b}{p_a} > 0,528 \\ 1, & \text{when } \frac{p_b}{p_a} \leq 0,528 \end{cases}, \quad (4)$$

$$G^* = 0,039 \cdot \mu \cdot b \frac{p_a}{\sqrt{\theta_a}} x_v, \quad (5)$$

where  $\mu$  – flow coefficient of throttle aperture;  $b$  – aperture width;  $\theta_a$  – input gas temperature.

But such approach reasonable only for approximate calculations because consider only one throttle aperture. There are following local resistances in the valve (see fig. 3): throttle apertures 1, flow in channel, throttle apertures 2.

Mass flow rate in compliance with (3)–(5) one can compute with following equation system:

$$\left\{ \begin{array}{l} G_1 = 0,039 \cdot \mu_1 \cdot b \frac{p_i}{\sqrt{\theta_i}} f\left(\frac{p_1}{p_i}\right) x_3, \\ G_2 = 0,039 \cdot \mu_2 \cdot b \frac{p_2}{\sqrt{\theta_2}} f\left(\frac{p_2}{p_i}\right) x_M, \\ \Delta p = p_1 - p_2, \\ G_1 = G_2 = G, \\ \Delta p = \xi \frac{v^2}{2} \rho, \\ v = \frac{G}{\rho A_{ch}}; \end{array} \right. \quad (6)$$

Here  $G_1$  and  $G_2$  – mass flow rates through first and second throttle apertures;  $A_{ch}$  – cross sectional area of the channel. Temperature could be accepted as a first approximation as a constant because changes in situations insignificantly. Pressure loss in valve channel  $\Delta p$  depends on gas velocity  $v$ , average density in channel  $\rho$  and local resistance coefficient  $\xi$  [11]. Local resistance coefficient depends on channel parameters, Reynold's number and gas flow type. Mostly coefficient value is within the limits of 0.15 and 2. By result of solution of equation system (6) mass flow rate through valve one can calculate:

$$\begin{aligned} G &= 0,039 \cdot \mu_1 \cdot b \frac{p_i}{\sqrt{\theta_i}} f\left(\frac{p_1}{p_i}\right) x_3, \\ p_1 &= \frac{1}{2} \left\{ p_i - \left(\frac{\mu_2}{\mu_1}\right)^2 \left(\frac{x_{max}}{x}\right)^2 \left(1 + 69\xi\mu_1^2 \frac{b^2 x^2}{A_{ch}^2}\right) p_o + \right. \\ &+ \left. \sqrt{\left(p_i - \left(\frac{\mu_2}{\mu_1}\right)^2 \left(\frac{x_{max}}{x}\right)^2 \left(1 + 69\xi\mu_1^2 \frac{b^2 x^2}{A_{ch}^2}\right) p_o\right)^2 + 4 \left(\frac{\mu_2}{\mu_1}\right)^2 \left(\frac{x_{max}}{x}\right)^2 \left(p_o + 69\xi\mu_1^2 \frac{b^2 x^2}{A_{ch}^2} p_i\right) p_o} \right\}; \end{aligned} \quad (7)$$

Flow coefficients  $\mu_1$  and  $\mu_2$  must be defined. Well known [9, 10] that coefficients values for throttle are within the limits of 0.6 and 1 and rise while aperture is opening. Flow coefficient  $\mu_2$  value equal 1 because aperture 2 is fully opened. Coefficient's  $\mu_2$  value decrease from 1 to 0.6 while aperture 2 is opening. Such situation takes place because gas flows through throttle window 1 into a narrow channel, and with increase of cross sectional area of the window its throughput doesn't rise.

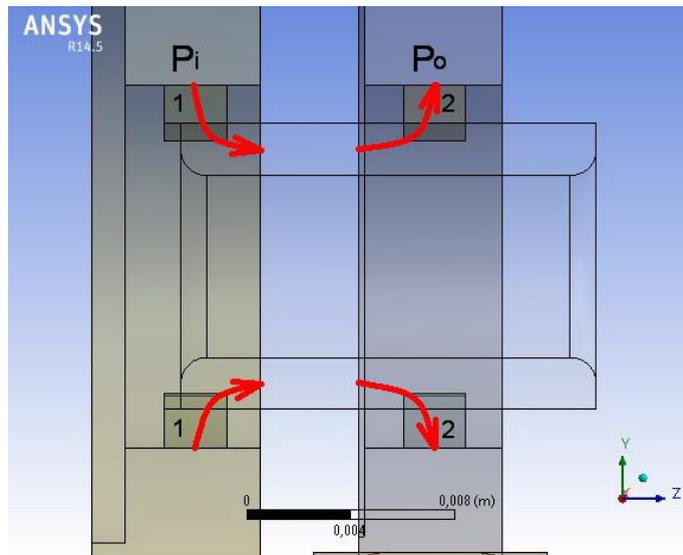


Fig. 3. Inner space longitudinal section.

To check theoretical results of mass flow rate the valve was simulated in ANSYS Fluent package. It is necessary to say few words about simulation parameters. For more adequate modelling results was used  $k-\omega$  SST (Shear Stress Transport) turbulent model [12]. But the most complicated was, firstly, construction of adequate mesh taken as a result significant calculations and time, and, secondly, many states being modelled and, as consequence, big work for systematization and analyses received results.

On fig. 4. are presented results of modelling of state with on 60% opened valve and input and output pressure of 5 and 3 bar. Figure 5a shows gas flow through the valve. It is clear that after two radial first throttle apertures two gas flows collide in channel making additional turbulence and flow resistance which decrease mass flow rate. On figure 5b is shown temperature varying  $\pm 5\%$ . This prove made earlier approximation.

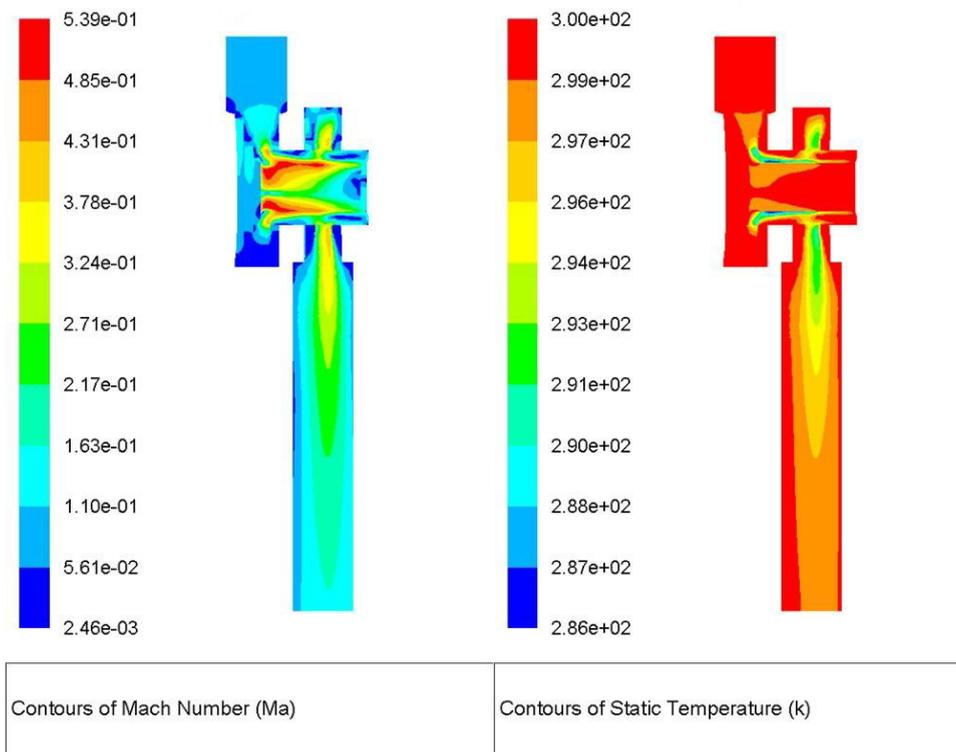


Fig. 4. Gas flow modelling results.

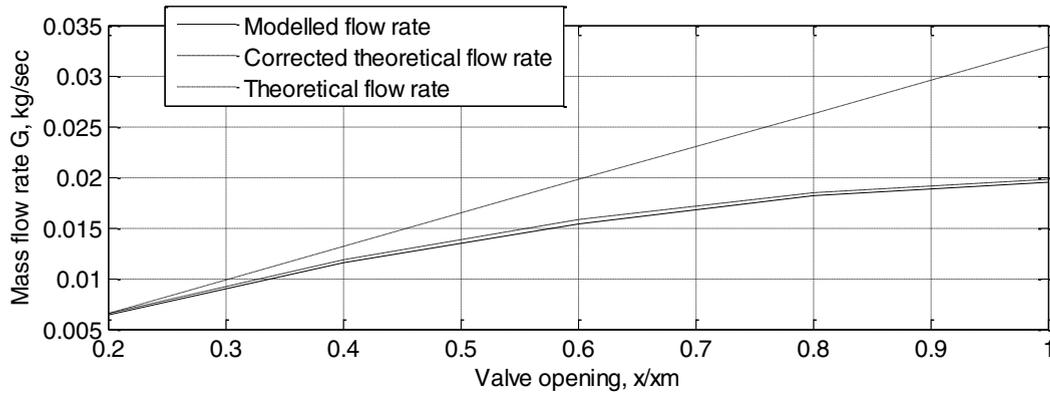


Fig. 5. Theoretical and modelled mas flow rates.

Let consider values of mass flow rate of the valve. On fig. 5 are shown theoretical mass flow rate and modelling results.

Received theoretical mass flow rate which consider elements of gas dynamics theory and modelling results differ slightly. It is clear that simplified valve model calculated with formulas

(3)– (5) could not give reliable value of mass flow rate through valve. One can draw the same conclusion analyzing diagrams of values of volume flow rate (see fig. 6) being calculated with formula (2).

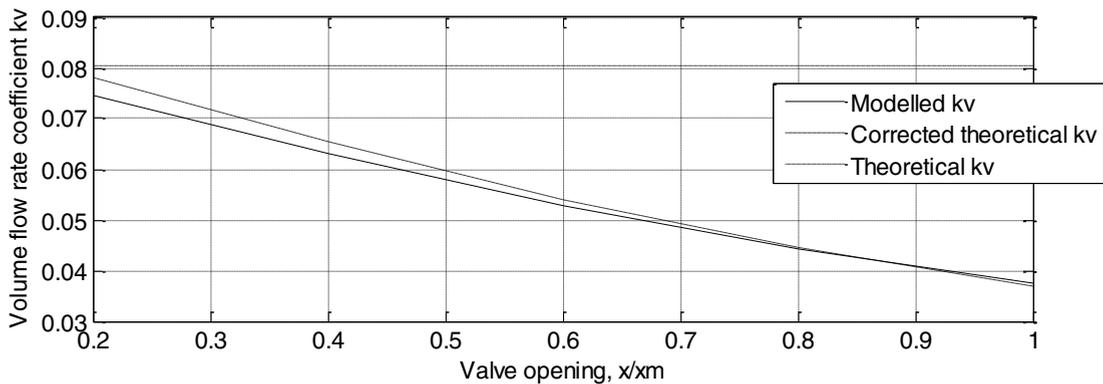


Fig. 6. Valve volume flow rate coefficient.

To make an intermediate total one can say that consideration of influences of peculiarities of compressed gas changes significantly mass and volume flow rates of the valve.

#### 4. Transfer function of the mechanical part

In the general case on the valve when moving have an effect following forces [10, 13]: inertial force, force of obliterated lock, friction forces, elastic forces and gas-dynamical forces. Obliterated lock force we will not considerate because appear in case of long stationary position of valve and body. Consideration of dry friction will insert significant nonlinearity in mechanical transfer function. But according to purpose of research it is necessary to investigate the influence of gas-dynamics forces. If it is necessary to considerate dry friction it is easier to insert a typical nonlinearity of dry friction.

Motion equation of the valve one can write:

$$m_v \frac{d^2x}{dt^2} = F_c - F_s - F_{fr} - F_{gd}, \tag{8}$$

where  $m_v$  – valve mass;  $F_c$  – control force;  $F_s$  – restoring spring force;  $F_{fr}$  – friction force;  $F_{gd}$  – gas-dynamics force influencing on the valve. From experiment [11] and modelling was identified that gas-dynamics forces are closing throttle aperture. On fig. 7 is shown gas flow scheme through valve.

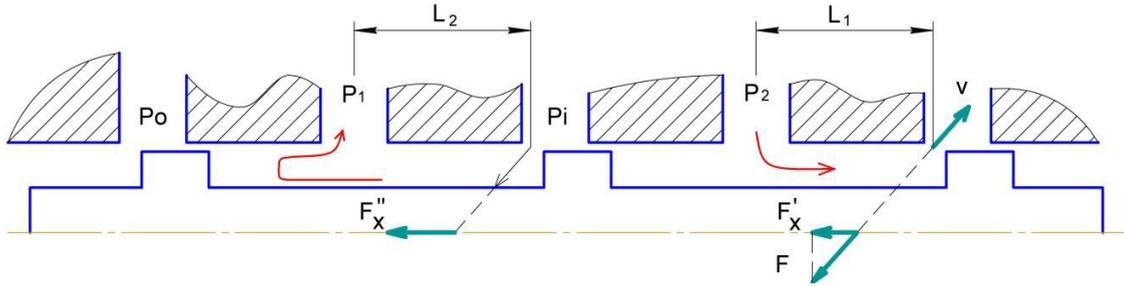


Fig. 7. Gas flow scheme through valve.

For the cell gas flows out of, axial component of gas-dynamics force one can calculate [11]:

$$F'_x = G_1 v_1 \cos\theta - L_1 \frac{dG_1}{dt}, \tag{9}$$

- where  $G_1 v_1$  – force of jet acting on valve’s cell;
- $L_1 (dG_1)/dt$  – force acting on the valve because of changing of velocity of gas mas in cell;
- $\theta$  – angle of gas flow, which is, according to Kirchhoff [8], 69°;
- $v$  – flow velocity in throttle aperture;
- $L_1, L_2$  – distance between edge and center of aperture.

For the cell gas flows in of, axial component of gas-dynamics is conditioned of flow braking in valve cell and momentum changing in cell, one can write:

$$F''_x = n_c G_2 v_2 + L_2 \frac{dG_2}{dt}, \tag{10}$$

where  $n_c$  – coefficient, which considerate changing of flow velocity because of changing its direction. Resultant gas-dynamics force axial component closing throttle aperture and can be calculated:

$$F_x = F'_x + F''_x = c_{gd}x + k_{in} \frac{dx}{dt}, \tag{11}$$

where  $c_{gd}$  – stiffness coefficient of “gas-dynamical spring”;  $k_{in}$  – coefficient that considerate inertial influence of transient gas flow on valve. So gas-dynamics force can be represented as spring with viscous friction.

It is necessary to evaluate maximal possible value of this force. It is clear that this force will be maximal when in both valve cells is critical flow. According to equations (9)–(11) and for simplicity assume mass flow rates and velocity in both cells equal, stiffness coefficient one can calculate:

$$c_{gd} = \frac{(n_c + \cos\theta) \cdot G \cdot v}{x} \approx \frac{0,465 \cdot G^* \cdot G^* R \theta}{x \cdot p S} = \frac{0,465 \cdot 0,0392 \cdot \mu^2 \cdot R \cdot b \cdot p \cdot x_3}{x} = 5,88 \cdot 10^{-3} \cdot \mu^2 \cdot b \cdot p; \tag{12}$$

It is also clear that inertial influence of transient flow on the valve will be maximal if take place critical flow and  $L_1 (dG_1)/dt = 0$ . Than

$$k_{in} = 0,039 \cdot \mu \cdot b \cdot \frac{p}{\sqrt{\theta}} \cdot L; \quad (13)$$

Now motion equation of the valve (8) one can write:

$$m_v \frac{d^2x}{dt^2} = F_c - k_{sp}x - f_{fr} \frac{dx}{dt} - c_{gd}x - k_{in} \frac{dx}{dt}; \quad (14)$$

As a result transfer function of mechanical part one can write:

$$W_m(s) = \frac{x(s)}{F_c(s)} = \frac{\frac{k_m}{k_{sp} + c_{gd}}}{\frac{m_v}{k_{sp} + c_{gd}}s^2 + \frac{f_{fr} + k_{in}}{k_{sp} + c_{gd}}s + 1};$$

In table 1 one can find values of time constants and damping coefficients of the valve with influence of gas-dynamical forces and without.

Table 1. Values of time constants and damping coefficients.

	Time constant $T_m, 10^{-3} \text{ sec}^{-1}$	Damping coefficient $\xi$
With influence of gas-dynamical forces	1,59028	0,5013
Without influence of gas-dynamical forces	1,59155	0,5000

Thereby it is possible to make an intermediate total that time constant and damping coefficient of transfer function of mechanical part almost do not depend on influence of gas-dynamical forces.

## Conclusion

Mathematical model of proportional electro-pneumatic spool valve was presented in this paper. The main idea of creation of such model was consideration of peculiarities of compressed gas on valve parameters. Development of the model included:

- General electro-pneumatic valve analysis
- Creation of transfer function of the valve (spool)
- Verifying received result using modelling
- Creation of transfer function of the mechanical part

Emphasis was made on influence of gas flows on model parameters and receiving universal mathematical model. Such model together with reliable models of forced membranate elements allow to receive more accurate pneumatic drive model.

The logical next step would be an including in received equations relations with varying temperature. Further research will be focused on verifying received results with help of experiment.

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