Abstract

The paper presents an approach for construction of optimal size ranges of modules for building technical products. The necessary input data are the demand (needs) function, defined for modules with known values of their main parameters, and the functional relationship between a chosen optimality criterion (total discounted costs for the service life of the size range) and the influencing factors. The approach includes a study of the optimality criterion's sensitivity towards the input data. The optimal size range provides minimum total costs while satisfying a known demand function. It is a compromise solution between the contradicting interests of manufacturers, for minimum number of sizes, and users, wanting large number of sizes. For finding the optimal size range, utilization of a suitable mathematical method for optimization is proposed, which method recognizes the characteristic features of the solved problem - discrete nature and large number of variants that have to be analyzed.

Keywords: design; technical products; size ranges; modules; optimization

1. Introduction

Because of its advantages the principle of modularity is widely used for designing and building modern technical systems [1, 3]. Most often the modules are elements of size ranges. Their creation is related to considerable initial investments, and the effectiveness of their application depends significantly on the diversity of the offered modules, which modules are characterized by a combination of values of their main parameters [4]. That is why achieving good economic results in the scope of production, as well as in the area of consumption, imposes precise and scientifically valid definition of the elements comprising the size ranges, i.e. of the values of their main parameters.

The analysis of known developments for choosing of an optimal size range of products with different functional
purpose shows absence of a general approach. Such an approach has to include, based on an analysis of the problem at hand, a systematization of the main problems, and, methods and means for their solution. Publications on the subject are mainly examining separate questions, while leaving some unsolved problems.

Mathematical models for solving problems with one parameter are presented in a number of publications [5, 6]. The presented “size range calculator” in [7] is applicable to this type of problems, too. Besides, with it can be studied size ranges with relatively small number of possible sizes. The use of models with multiple parameters is limited in practice.

Another disadvantage in the known developments on the subject is related to the choice of optimality criterion. As such most often are used the discounted (total) costs, but the existing methods take into account different components of these costs [5, 6]. Usually interest costs, devaluation of national currency, batch volume of the manufactured modules, learning curves, etc. are not considered. Most fully the latter factors are taken into account in [8], but in it only production costs and costs for designing every size are taken into consideration. In other developments, the optimality criterion includes the level of conformity of the chosen size range variant with a size range determined through market research [9]. This approach in principle is correct, but with limited use, because determining the economic outcomes from the inconformity between demand and supply is related to a number of difficulties and unsolved issues. In [7] the optimality criterion used is direct and indirect costs. A limited number of functional models are proposed for the determination of the direct and indirect costs.

In some developments the demand for different sizes is not considered in evident form, i.e. the demand function [1]. Furthermore, in many cases the mathematical methods for optimization are inappropriate and/or inefficient. This is not only due to lack of information and/or limited access to the newest methods, but also due to the sometimes complex mathematical apparatus and “unfriendly” user interface of the software products used by the designers of technical systems. The developments regarding design and choice of an optimal size range are “know-how” for companies occupied with such activity, and scientific research teams specializing in this domain aim to sell the achieved results as special developments to interested manufacturers at a high price.

The purpose of this paper is analysis of the problem for size ranges optimization, determination of its characteristic features and on this basis a proposition of an approach for solving the problem.

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2. Formulation and analysis of the problem for choosing of an optimal size range

In consideration are modules $z$, each of which represents combination of values of chosen parameters:

$$ z = \{ x_1, x_2, \ldots, x_w, \ldots, x_W \} $$
As a result from statistical processing of market research data, the modules’ demand function 
\( N(z) = N(x_1, \ldots, x_w, \ldots, x_W) \) is determined according to the values of the modules’ parameters, and for every parameter:

\[ x_w \in \bar{X}_w = \{ x_{w,1}, x_{w,2}, \ldots, x_{w,m_w}, \ldots, x_{w,\bar{M}_w} \}, \quad w = 1 + W, \quad m_w = 1 + \bar{M}_w, \]

In this case the demand function is satisfied by the size range \( \bar{Z} = \{ \bar{z}^1, \bar{z}^2, \ldots, \bar{z}^l, \ldots, \bar{z}^L \}, \quad l = 1 + \bar{L} \). Obviously this number is very big. For reducing the number of sizes the values \( x_w \) of the modules’ parameters are chosen according to standards or ranges of preferred numbers, i.e.:

\[ x_w \in \bar{X}_w = \{ x_{w,1}, x_{w,2}, \ldots, x_{w,m_w}, \ldots, x_{w,\bar{M}_w} \}, \quad w = 1 + W, \quad m_w = 1 + \bar{M}_w \]

We assume that every size \( \bar{z}^l \), \( l = 1 + \bar{L} \), is used for satisfying the demand for modules with values of their parameters in the range from \( \bar{x}^{-l}_w \) to \( \bar{x}^l_w \), where \( \bar{x}^{-l}_w \in \bar{X}_w, \quad \bar{x}^l_w \in \bar{X}_w, \quad \bar{x}^{-l}_w < \bar{x}^l_w, \quad w = 1 + W \). Therefore the demand for every size \( \bar{z}^l \), \( l = 1 + \bar{L} \), is:

\[
N^l = \int_{\bar{x}^{-l}_w}^{\bar{x}^l_w} \cdots \int_{\bar{x}^{-l}_w}^{\bar{x}^l_w} N(x_1, x_2, \ldots, x_W) dx_1 dx_2 \ldots dx_W ,
\]

where \( N(x_1, \ldots, x_w, \ldots, x_W) \) is the differential demand function.

In this case, also, the number \( \bar{L} \) of elements in the size range \( \bar{Z} = \{ \bar{z}^1, \bar{z}^2, \ldots, \bar{z}^l, \ldots, \bar{z}^L \}, \quad l = 1 + \bar{L} \), satisfying the demand for modules with standard values of their main parameters is very big, and few manufacturers could afford to produce all sizes. For example, only for one of their main parameters – load capacity, linear modules for building industrial robots are scores in term of sizes - 0.02; 0.05; 0.08; 0.1; 0.16; 0.2; 0.32; 0.4; 0.5; 0.63; 0.8; 1.0; 1.25; 1.5; 1.6; 2.0; 2.5; 3.2; 4.0; 4.2; 5.0; 6.0; 6.3; 8.0; 10; 12; 15; 16; 20; 25; 32; 40; 45; 50; 60; 63; 80; 100; 120; 125; 150; 160; 180; 200; 250; 320 [kg], etc. This number significantly increases when taking into account other parameters of the linear modules – length of stroke, velocity, positioning accuracy, etc., which also can take different values.

When there are no constraints regarding the combination of values of the main parameters, the number of elements in the range \( \bar{Z} = \{ \bar{z}^1, \bar{z}^2, \ldots, \bar{z}^l, \ldots, \bar{z}^L \} \) is \( \bar{L} = \prod_{l=1}^{W} \bar{M}_w \). It must be noted, that in every range, when solving the problem, it is obligatory that a module with maximum values of its main parameters is included, i.e. \( z^L = \{ x_{1,\bar{M}_1}, x_{2,\bar{M}_2}, \ldots, x_{w,\bar{M}_w}, \ldots, x_{W,\bar{M}_W} \} \), which is due to the requirement for satisfaction of the demands of all customers.

On Fig. 1 is presented the process of reducing the number of sizes through the use of ranges with preferred numbers. The example is for \( W = 1 \).

In the general case, size ranges \( \hat{Z} = \{ \hat{z}^1, \hat{z}^2, \ldots, \hat{z}^l, \ldots, \hat{z}^L \} \) with limited number of sizes \( \hat{L} \) are offered on the market, where typically \( \hat{L} \leq \bar{L} \).

This inequality expresses the opposing interests of users and manufacturers. For the users of the product it is more advantageous that the maximum number of sizes is offered on the market, i.e. the condition \( \hat{L} \approx \bar{L} \) is fulfilled. In that way their needs for modules with certain values of the parameters are fully satisfied, and losses (additional costs) due to discrepancy between demand and supply are minimized. But in this case the manufactured products will have greater prime cost, because batch volume decreases for each size. As a result, it is possible that the total costs increase, and the economic efficiency of the size range’s application decreases.

On the other hand, as strict the inequality \( \hat{L} \leq \bar{L} \) is, as much increases the batch volume for each manufactured size, and therefore the possibilities for lowering prime costs increase. But this eventually will elevate the costs for
users due to increased discrepancy between needs for modules with certain values of their parameters and the modules offered on the market. Therefore, modules with overdesigned characteristics will be used. In this case, also, the application of the designed size range will be under question.

Consequently, for solving this contradiction a size range of modules fulfilling the demand function has to be found, which is a compromise between the users’ requirements for greater number of sizes (less overdesign) in the size range, and the requirements of the manufacturers for lesser number of sizes and bigger batch volumes (lower prime costs).

An idea for the nature of this compromise, with regard to the number of elements in the size ranges, is shown on Fig. 2, where curve 1 depicts costs for use (exploitation) of the size ranges, curve 2 – costs for manufacturing of the size ranges, and curve 3 – total costs as in manufacturing and exploitation. Every point on the abscissa is a size range with certain number of elements which satisfies all needs. The ordinates of curve 3 are equal to the sum of the ordinates of curves 1 and 2. As is seen on the figure, the total costs are minimum in point $L'$. The latter corresponds to a size range with optimum number of sizes.
Finding the optimal size range is related to solving the following problem:

With respect to the condition for satisfying the needs for products, which condition is given by a demand function, determine the number of elements in the size range, the values of their main parameters and the necessary production volume for each size, so that a predefined criterion including the financial (economic) interests of both users and manufacturers have optimal (minimum) value.

The analysis of the defined problem reveals some of its characteristic features, which have to be taken into account when formalizing the problem and choosing suitable methods for optimization.

1. In the general case the defined problem is with multiple parameters, since every product is characterized by a number of parameters some of which are main parameters, and others – secondary parameters. The main parameters of the product characterize its capability for executing a set of given functions. All the rest parameters, which have secondary importance for the product and do not determine its working capacity, are secondary.

2. The defined problem belongs to the class of problems of the discrete programming. Finding a solution is accompanied by significant difficulties. Solving the discrete problem by replacing it with a continuous analogue and consequent rounding of the obtained solution to an integral one is impossible. By its nature the problem refers to distribution problems, but differs from them by its variable number of arguments and variable argument values, which complicates its solution.

3. The great diversity of main parameters and the great number of their possible values determine the availability of a great number of variants for size ranges that have to be evaluated. For example for the simplest single parameter problem, \( W = 1 \), the number of possible variants for size ranges including \( L \) number of elements from \( L \) possible is \( \binom{L}{L} \). In this case, for the analysis of all possible ranges it is necessary to consider \( \sum_{L=1}^{T} \binom{L}{L-1} = 2^T - 1 \) variants [5]. The number of variants increases significantly for problems with multiple parameters, and even the use of modern computer technology is inefficient and/or impossible. Therefore, the choice of an optimal size range is related to considerable number of calculation procedures. That is why the use of a method for directed search of the optimal solution is advisable.

4. The solution of the problem significantly depends on the chosen optimality criterion. The latter determines the solution’s properties. When developing the optimality criterion (objective function) it is necessary to consider the described contradiction between the interests of users and manufacturers.

5. For determining an optimal size range it is necessary to make a prognosis for the needed modules and to define a functional relationship between the chosen optimality criterion and the influencing factors. The solution of these problems is related to collecting and processing of a great amount of technical and economic data, including the relationships between the designed modular system and the environment throughout the different stages of the system’s lifecycle. This involves significant expenses in time and labor.
of highly qualified specialists. Moreover, the described activities are accomplished under the conditions of uncertainty and incomplete information. Therefore it is necessary to provide procedure for determination of the influence of the input data over the obtained solution and to determine the data for which the most precise and reliable information has to be collected.

3. Approach for choosing of an optimal size range

From the conducted analysis it can be concluded that choosing an optimal size range is a complex engineering and economic problem. In addition, the problem is also interesting from a mathematical standpoint. For its solution an approach is proposed consisting of the following main stages:

Stage 1. Choice of main parameters of the modules.
Stage 2. Determination of the needs and building the demand function.
Stage 3. Choice of optimality criterion.
Stage 4. Building a mathematical model.
Stage 5. Determination of the functional relationship between the optimality criterion and the influencing factors.
Stage 6. Study of the optimality criterion’s sensitivity.
Stage 7. Choice of mathematical method.
Stage 8. Algorithmic and programming support.
Stage 9. Solving the problem.

We will lay out, in short, the essence of some of the main stages of the proposed approach.

3.1. Choice of main parameters of the modules – Stage 1

The expedient determination of the combination of main parameters, according to which the optimization will be performed, predetermines to a great extent the effectiveness of the size ranges. The chosen main parameters must correspond to the following conditions [5]: characterize to the greatest extent the technical, exploitation and technological capabilities of the products; to be independent from frequently changing factors such as manufacturing technology, materials used, etc.; not to constraint the possibilities for design improvements, etc.

The values of the product’s parameters are regulated by norms, standards or are chosen from the elements of ranges with preferred numbers – $R_5$, $R_{10}$, $R_{20}$, $R_{40}$, and $R_{80}$. The members of every range are rounded values of geometric progressions with base $a = 1$, and exponent $q$ equal respectively to $\sqrt[5]{10}$, $\sqrt[10]{10}$, $\sqrt[20]{10}$, $\sqrt[40]{10}$, and $\sqrt[80]{10}$. The exponent characterizes the step (density) of the ranges.

At this stage the step of the main parameters is determined, too. It can be determined through studying of the system “market – design – manufacturing – marketing” [1]. Very often the use of ranges with the same density is in contradiction with the market requirements. In these cases the whole range can be broken into sub-ranges which use ranges with different values of the exponent $q$, i.e. with smaller ($R_5$) or bigger density ($R_{10}$) [1].

3.2. Determination of the needs and building the demand function – Stage 2

For determination of the needs for products, data is used that is obtained from sales, including e-trading, client inquiries, literature sources, inquiring and investigation of potential users, previous experience, market experiments, information systems for consumption watch, etc. After gathering the input information the latter has to be processed with the use of suitable methods. For modelling the needs for modules it is necessary to apply methods for demand forecasting and approximation. The forecasting methods are divided into two groups: methods based on qualitative evaluations, and methods based on quantitative data. The first group is based on expert evaluations and includes the following basic methods: virtual markets [10], method Delphi [11], game theory [12], general analysis [13], etc. The following methods relate to the second group: extrapolation, neural networks [14], data mining [15], causal model [16], etc.

The task of collecting and processing the data regarding the needs for products is very expensive, requires profound knowledge of methods with complex mathematical apparatus, relates to considerable amount of calculation procedures, and its solution is impossible without the use of suitable software products.

Most often the needs for different sizes of products are given by a demand function
\( N(z) = N(x_1, \ldots, x_n, \ldots, x_W) \) and represent the number of requests in corresponding regions in the \( W \)-dimensional Euclidean space. These regions are multidimensional parallelepipeds with orthogonal surfaces. The vertices of these parallelepipeds are produced by intersection of surfaces that are perpendicular to the corresponding coordinate axes and pass through points \( x_{w,1}, x_{w,2}, \ldots, x_{w,m_w}, \ldots, x_{w,\eta_w}, w = 1 + W \). The needs for modules can be given as a differential or integral function [5] or in table (discrete) form.

For example, for \( W = 2 \) the demand function for modules with standard values of their parameters, in table form, is shown in Table 1, where: \( N_{m_1,m_2} \) - need for products with values of their main parameters in the limits \( x_{1, \ldots, m_1} + x_{1, m_1} \) and \( x_{2, \ldots, m_2} + x_{2, m_2} \).

### Table 1. Demand function.

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#### 3.3. Choice of optimality criterion – Stage 3

The optimality criterion is used for evaluation of the feasible solutions, which is why the choice of optimality criterion predetermines to a great extent the quality of the solution. The choice of suitable mathematical method for solving the problem is also dependent on the optimality criterion. The simplification of the optimality criterion allows for building a mathematical model with desired characteristics such as linearity, continuity, etc. At the same time oversimplification leads to imprecise, incomplete description of the problem, and as a consequence lowers the quality of the found solutions. On the other hand, choosing a very complex optimality criterion leads to a nonlinear and discrete mathematical model, that will make difficult the choice of solution method, and in some cases make impossible the finding of a solution [9]. Therefore the choice of an optimality criterion is a compromise between these two approaches.

As a criterion for evaluation of the different size ranges the use of the total discounted costs for the period of exploitation is proposed [17].

\[
R = K_{WB} + (K_A + K_Z + K_I + K_R + K_E) \cdot D
\]

where [18]:

\[
K_{WB} = K_A (1 + a)^n, \quad D = \frac{(1 + a)^n - 1}{a}, \quad K_A = \frac{K_{WB}}{n}, \quad K_Z = \frac{K_{WB}}{2}, K_I = K_{WB} \cdot F,
\]

\( K_{WB} \) is the capital investments needed for buying the size range of modules with regard to the factor “time”; these investments are used as an evaluation of the costs in the scope of manufacturing; \( K \) - price of the products; \( K_A, K_Z, K_I, K_R, K_E \) - corresponding annual costs for depreciation, interests, maintenance and repair, floor space and energy; \( D \) - discount coefficient of the exploitation costs for the whole exploitation period of the size range; \( a \) - discount coefficient of the capital investments; \( n \) - exploitation period, yrs.; \( F \) - coefficient regarding costs for maintenance and repair during exploitation.

When determining the price \( K \) of the products, the size of the production program has to be taken into account. The decrease in price due to increase in the number of the manufactured products is determined by [19]:
- costs for design and organization of the manufacturing process are distributed among greater number of products;
- when manufacturing great numbers of similar products possibilities for improvement of the technological process arise. This leads to lowering the costs for manufacturing and to savings from materials due to their more rational use;
- increasing the manufacturing program is related to building up manufacturing experience, quality improvement, lowering waste and increase in productivity;
- with the buildup of manufacturing experience possibilities are created for improvement of the structural design of the construction.

For taking into account the increase in productivity due to experience buildup (learning) the so called learning curves are used. They facilitate determination of manufacturing costs [20, 21]. For mathematical modeling of the learning processes different models are used. Some of the most commonly used are log-linear, hyperbolic, and exponential model [22]. The common between these three models is that they are used for learning process including only one variable. For complex processes which have to be described with better precision models including multiple variables are used.

For taking into account the influence of the modules’ production volume over their price $K$ an exponential dependency of the following kind is proposed:

$$K(N^l) = K(N) \cdot \left( \frac{N^l}{N} \right)^v$$

where $K(N^l)$ is the price of the $l^{th}$ size for production program $N^l$; $K(N)$ - price of the $l^{th}$ size for production program $N$; $v$ - coefficient characterizing the change rate of the modules’ price in relationship to change in the production program.

3.4. Building a mathematical model – Stage 4

The mathematical model of the defined problem for $W = 2$ is [17]:

Find $L^*, Z^* = \{z^1, z^2, \ldots, z^l, \ldots, z^w^*\}$, $N^1, N^2, \ldots, N^v$, $N^l$, such that the total discounted costs have minimum value:

$$\text{min } R = f(L, z^1, \ldots, z^l, N^1, \ldots, N^l) = \sum_{l=1}^{L^*} f_1(z^l, N^l) = \sum_{l=1}^{L^*} f_2(x^l_1, x^l_2, N^l)$$

with the conditions:

$$x^1_1 \in \bar{X}_1 = \{\bar{x}_{1,1}, \bar{x}_{1,2}, \ldots, \bar{x}_{1,m_1}, \ldots, \bar{x}_{1,M_1} \}$$

$$x^1_2 \in \bar{X}_2 = \{\bar{x}_{2,1}, \bar{x}_{2,2}, \ldots, \bar{x}_{2,m_2}, \ldots, \bar{x}_{2,M_2} \}$$

$$\sum_{l=1}^{L^*} N^l = \sum_{m_1=1}^{M_1} \sum_{m_2=1}^{M_2} N_{m_1,m_2} = N_0$$

The assumption is made that each size $z^l = \{\bar{x}_{1,m_1}, \bar{x}_{2,m_2} \}$ can perform the functions of all modules with values of their main parameters in the ranges $0 \div \bar{x}_{1,m_1}$ and $0 \div \bar{x}_{2,m_2}$.
3.5. Choice of a mathematical method – Stage 7

The size range is a complex technical and economic system, including multitude of elements (sizes) and cause-effect relationships between them. Therefore the problem for size range optimization can be seen as a multistage process for management of a complex system. This process is such that in every separate stage a decision has to be made which decision is optimal for the whole process under consideration. The goal for the decision making in every stage is to minimize the total discounted costs that are expressed through the chosen optimality criterion, and to determine the values of the control parameters. In the problem under consideration the control parameters are the sizes included in the size range and their corresponding production program.

One of the efficient methods for solving of this class of problems is the dynamic programming method. The method is based on the optimality principle of R. Belman. On the basis of this principle for $W = 2$ the following recurrent dependency for determination of the total discounted costs is obtained [17]:

$$
R_{m_1, m_2}^I = \min \left\{ R_{m_1, m_2}^{I-1} + f_2(\bar{x}_{1,m_1}, \bar{x}_{2,m_2}, \sum_{p=1}^{m_1} \sum_{q=1}^{m_2} N_{p,q} - \sum_{z=1}^{m_1} \sum_{l=1}^{m_2} N_{z,l}) \right\}
$$

$$
m_1' \in [1,m_1], m_2' \in [1,m_2]
$$

$$
m_1 + m_2 \in [l, m_1 + m_2]
$$

where $R_{m_1, m_2}^I$ is the minimum total costs for satisfying the needs for modules having values of their main parameters $\bar{x}_{1,m_1}$ and $\bar{x}_{2,m_2}$ with $l$ number of sizes.

The calculation of $R_{m_1, m_2}^I$ continues until the following condition is met:

$$
R_{m_1, m_2}^I \leq R_{m_1', m_2'}^I
$$

Conclusion

The problem for choosing an optimal size range is defined. On the basis of an analysis its characteristic features are determined. These characteristic features have to be taken into consideration when the problem is formalized and when the choice of an optimization method is made. An approach is proposed for solving the defined problem. The approach consists of nine stages. A description is made of the main stages which description includes the problems solved and the means with which to solve them. These means include: recommended methods for determination of the needs for products, choice of a criterion for evaluation of the size ranges, which criterion also takes into account the production program for the manufactured products and the built up experience, a mathematical model of the problem, choice of a suitable mathematical method for solving the problem, and a recurrent dependency obtained for calculation of the total costs.

The presented approach has to be taken as a “plan of action”, and its stages have to be analysed thoroughly and customized where needed. For each stage the tasks and problems solved have to be precisely formulated and adequate means for their solving have to be offered. In this line of thought, the proposed approach offers a great opportunity for future research, including study and analysis of known developments related to the described problems in the different stages, development of new and/or improved models, methods, algorithms and software applications, experimental research, etc.

The application of the proposed approach would lead to lowering the exploitation and production costs of the modular technical systems. According to a preliminary calculation of the authors this lowering could be in the range between 15-30%, depending on the studied subject, and the conditions of its production and exploitation.

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