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## Investigation of the Dynamic Behaviour of CFRP Leaf Springs

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### Abstract

The paper deals with the dynamic behaviour of CFRP leaf springs. The experimental modal analysis is used for the investigation. Therefore two different methods (impact test and shaker test) for the excitation of the parts have been applied. A standard steel spring is used as reference. Three different composite springs are investigated and compared. The composite design is calculated via classical lamination theory and the manufacturing was done by hand lay-up and autoclave. An important aspect for spring elements is the behaviour at different thermal conditions. Therefore one of the test bodies was investigated at low temperature. At last the suitability of the analytical calculation of the Euler – Bernoulli beam theory was investigated and compared with the experimental results.

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### 1. Introduction

The light weight engineering becomes more and more important. Especially rising energy cost and the international growing request to save the environment with the help of optimised usage of resources and the reduction of emissions is determining the constantly increasing demand of efficient or high performance products. The use of fibre reinforced polymers is perhaps the most promising technology in this field. Such materials like carbon fibre reinforced polymers (CFRP) or glass fibre reinforced polymers (GFRP) are used in many different branches for example the aerospace industry, the energy industry or the transportation industry. Most applications made from

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these materials have the requirement that their shape needs to be constant under load. For this behaviour the stiffness is the most reasonable parameter. In some other applications, like in spring elements or in the usage in a plane's wing, the mechanical strength and the tensibility are also major characteristics. For these fields of application the knowledge about the dynamic behaviour becomes as important as the static behaviour. In this work the investigation of the dynamic behaviour of composite leaf springs shall reveal the potential of such products. Another novelty for leaf springs is the use of CFRP instead of GFRP, which is the standard work piece material for this type of application. This situation is based on the higher breaking strain of glass fibres and the associated higher travel of the spring. Higher stiffness of carbon fibres would reduce the volume of the part in comparison to glass fibre thus leading to less weight. An important aspect is that except of the price for the raw material, there are no additional expenses for the processing of carbon fibres instead of glass fibres.

## 2. Construction of the specimen

The tested spring elements are produced for the railway industry. The reference spring is made of spring steel 1.8159 (51CrV4) and is painted to avoid corrosion during the usage. The elimination of this coating process would be an advantage for the composite spring, because neither the fibres nor the resin matrix system of such a compound is susceptible to corrosion. In table 1 an overview of the properties of the steel spring and three specimens of composite springs is given. The composite springs are made of unidirectional material and fabric layers. In detail the unidirectional layers are prepreg with high strength carbon fibre and an epoxy resin whereas the fabric layers are plain woven prepreg with standard carbon fibre also using an epoxy resin. The differentiation in these two materials provides optimal orientated fibres to absorb the load through the spring work and better impact behaviour through the woven fibre architecture. The layup of the springs was done manually. For the consolidation a specific pressure and temperature cycle was accomplished in the autoclave. When constructing the springs, classical lamination theory, with the help of correction values based on [1] was applied.

Table 1. Properties of the specimen; CF-E = carbon fibre epoxy compound.

	dimensions l x w x h [mm]	moment of inertia [mm <sup>4</sup> ]	material	mass density [kg/m <sup>3</sup> ]	young's modulus [GPa]	bending rigidity [Nm <sup>2</sup> ]	weight [g]
Steel	297 x 25 x 2,2	22,2	steel	7850	210	4,662	126,5
CFRP1	297 x 25 x 2,9	50,81	CF-E	1515,6	167	9,654	32,25
CFRP2	297 x 25x 2,3	25,35	CF-E	1518,7	118	4,817	25,63
CFRP3	297 x 25 x 1,9	14,29	CF-E	1518,5	146	2,715	21,17

## 3. Dynamic behaviour

The experimental modal analysis delivers the modal parameters, modal frequencies, mode shapes and modal damping of parts [2]. The measured frequency response function (FRF) is the compliance, due to the used displacement transducer, therefore the calculated amplitudes of the mode shapes are in units of  $\mu\text{m}/\text{N}$ . In this work the excitation of the structure was done in different ways to determine which technique works best and conducts an accurate testing. The experimental investigation was performed by measuring one single column of the transfer matrix [3]. In this case a single-input single-output (SISO) modal testing was done. This can be achieved by roving the displacement sensor and retain the point of excitation for each FRF measurement on the same position. Therefore the force of excitation is stationary. The decision for measuring a single column of the transfer matrix was made due to the delicate structure of the leaf springs. A roving hammer impact causes double hits on the surface and fudged the response signal. Different methods were performed to calculate the frequency response function (FRF) for each measuring point.

### 3.1. Impact testing

The first excitation of the leaf springs was done by using an impact hammer with a built in force transducer for the reference signal and a roving triaxial acceleration transducer with a mass of 2.9 g. The influence of the roving mass of the accelerometer leads to a wide distributed range of the single amplitude peaks in the transfer function. For the steel leaf spring the peak distribution was at the 2<sup>nd</sup> (12 Hz), at the 3<sup>rd</sup> (24 Hz) and the 4<sup>th</sup> Mode (56 Hz) for each point. Due to this fact, the measurement has to be done by using a none contact displacement sensor which does not influence the mass distribution of the system. Therefore the response signal was captured with a none contact eddy current displacement sensor, namely Micro Epsilon Eddy NCDT 3300. The sensor has a measuring range of 0-1 mm. To enhance the functionality of the eddy current sensor, a conductive surface was needed. Therefore an aluminum foil (0.01 mm) was applied on the upper surface of the CFRP leaf springs. The used hammer tip for the excitation influences the impulse shape and the bandwidth of the excitation [4]. In this case a tip made of polymer (delrin) was used.

### 3.2. Shaker testing

A swept sinusoidal signal over a time period of 10 s was applied to excite the structure within a range of 10 Hz to 2 kHz. The used shaker was the modal exciter 4826 from Brüel & Kjær. By averaging over three periods for each point the response signal of the leaf spring was calculated. The response signal was already captured with a non contact eddy current displacement sensor.

### 3.3. Experimental set up

As described above, two different methods to excite the structure were applied. In figure 2 the measurement setup for both methods is shown. The leaf spring (1) is mounted as cantilever beam on a horizontal socket (2). Further the socket is fixed on a cast iron table with clamping claws (3). The clamped length of the spring was 40 mm. Mounting was done via predefined screws and bores at the part. Changes in the physical properties, such as the reduction in stiffness resulting from cracks or delamination, would lead to measurable changes in the modal properties [5]. The contactless eddy current displacement sensor is attached on a hydraulic gauge holder (4). The excitation of the structure is done by impact testing with an impact hammer (5) and an attached load cell (6) shown in figure 1, a. In figure 1, b the measurement setup for the shaker testing is given. In contrast to sinusoidal excitement by using a shaker (figure. 1, b), a broadband excitation by an approximate dirac delta function can be achieved by impact testing [2]. For the shaker testing the load cell was mounted as close as possible to the structure (figure. 1, b).

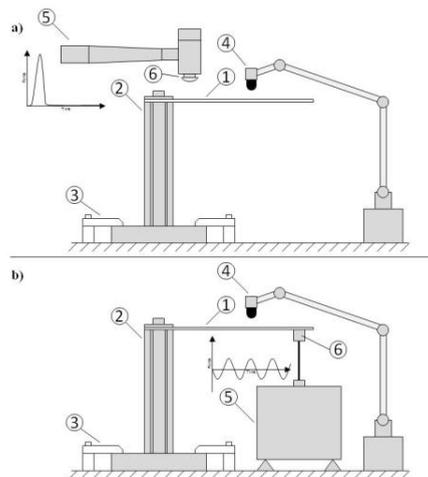


Fig. 1. (a) impact testing; (b) shaker testing.

#### 4. Data processing

To describe the physical properties of the system, the modal parameters have to be estimated from all collected data of the measured structure. The measured data were recorded in the time domain. The response time is a measure of displacement with the unit  $\mu\text{m}$  and the input signal is a measure of force with the unit N. By using the software LabView the transfer function of the type  $\mu\text{m}/\text{N}$  was calculated. In the case of the impact testing the FRF was directly calculated by a force triggered displacement eddy current sensor. Therefore the frequency transformed data of the displacement were divided by the frequency transformed data of the excitation force. Additionally an averaging of the calculated FRF was performed. For the calculation of the shaker generated FRF a post processing was needed and the triggering of the time signals have to be done manually. In both cases, the result of the FRF is also known as the compliance [3].

#### 5. Results of the investigation

The Complex Mode Indicator Function (CMIF) was used to identify the number of eigenfrequencies in the investigated frequency range. An evaluation of the damping properties of the leaf springs for each vibration mode for each single point of the investigated body was carried out. Therefore the Rational Fraction Polynomial Method has been used to analyse the system in the frequency domain [6], [7]. These methods have been performed by using the software package ME scope.

Table 2 shows the results of the shaker testing based on the resonance frequency and the estimated damping parameters. For this purpose the original steel body is taken as reference for the identified parameter. All three CFRP bodies have higher damping properties in the first three resonance frequencies. Low masses of the components lead to higher resonance frequencies [8], which fits to the measured data of the CFRP bodies.

Table 2. Results of the shaker testing.

mode shape	Steel		CFRP1		CFRP2		CFRP3	
	frequency [Hz]	damping [Hz]						
1 <sup>st</sup>	24,3	0,07	61,2	0,21	42,5	0,29	37,94	0,15
2 <sup>nd</sup>	145	1,26	353	5,04	252,7	9,08	228,8	1,74
3 <sup>rd</sup>	367	5,8	720	22,6	301,3	9,36	302,4	10,5
4 <sup>th</sup>	598	15,9	1170	7,56	481,2	33,1	459	20,4
5 <sup>th</sup>	962	3,56	-	-	576,6	36,4	590,8	41,8
6 <sup>th</sup>	1500	8,64	-	-	824,3	8,58	740,1	10,2
7 <sup>th</sup>	-	-	-	-	1493	27,9	1328	7,73

Figure 2 shows the mode shapes of the investigated leaf springs with shaker excitation. Due to the uniaxial eddy current displacement sensor, only the bending modes were identified. Hence, only the modal parameters within the excited frequency range of 10 Hz and 2 kHz can be captured. The leaf spring CFRP1 only delivers a sufficiently detailed signal up to 1200 Hz and only mode shapes can be displayed.

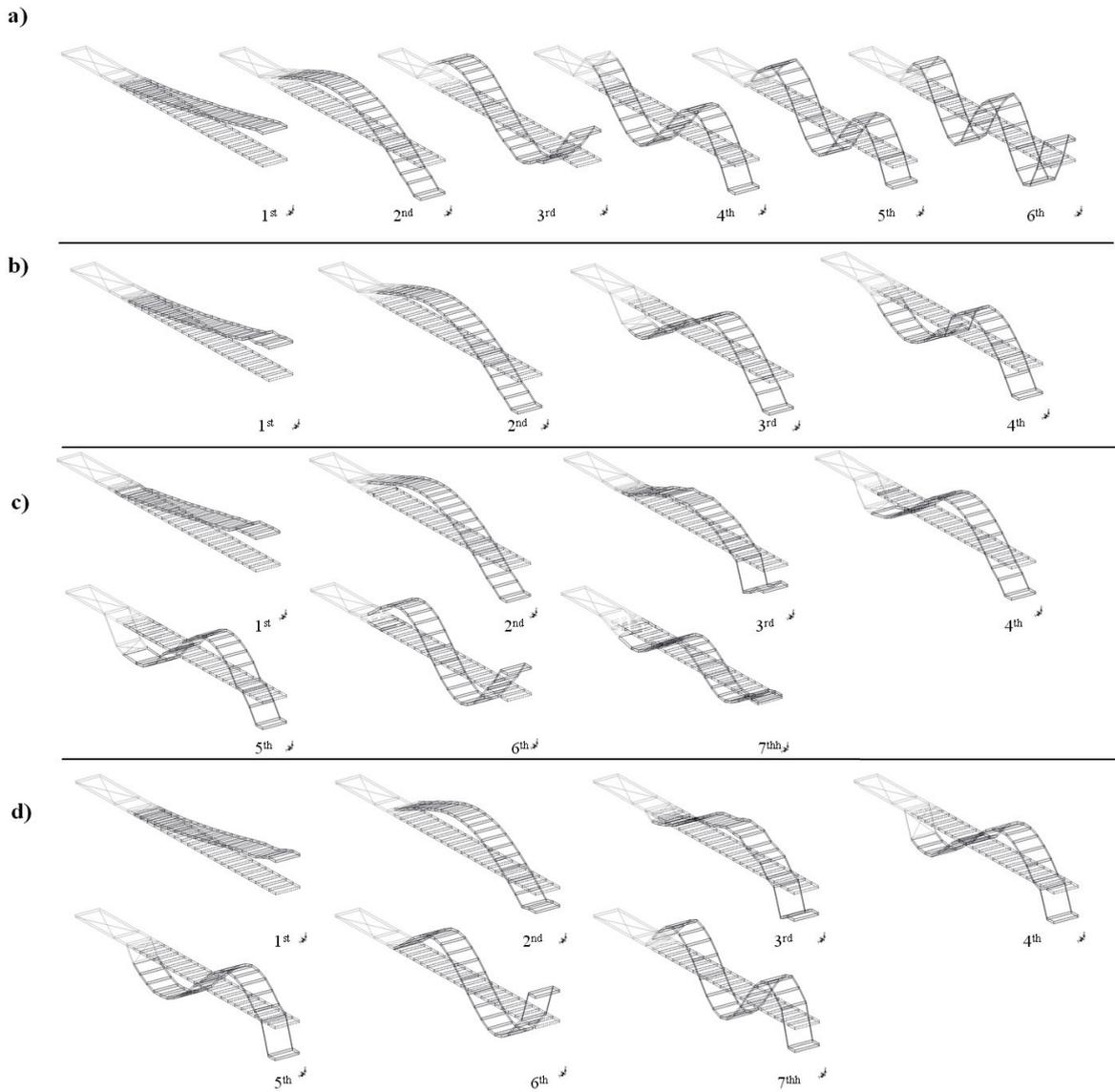


Fig. 2. Mode shapes at resonance frequencies of the shaker test; (a) Steel; (b) CFRP1; (c) CFRP2; (d) CFRP3.

In order to determine the influence of the temperature, the CFRP1 leaf spring was tested under different environmental conditions. The standard condition is the test temperature of 23 °C. Alternatively the leaf spring was tested at -8 °C. Such a temperature change effect is especially useful and very important for the railway industry. The excitation was - done by the shaker in the same frequency range as the testing under standard conditions. See table 3 for the results.

Table 3. Temperature influence on leaf spring CFRP1.

mode shape	CFRP1 at 23°C		CFRP1 at -8°C	
	frequency Hz	damping Hz	frequency Hz	damping Hz
1 <sup>st</sup>	61,2	0,105	58,4	3,38
2 <sup>nd</sup>	353,0	2,52	60,8	1,04
3 <sup>rd</sup>	720,1	11,31	345,0	8,06
4 <sup>th</sup>	1170,0	3,78	638,6	53,4
5 <sup>th</sup>	-	-	823,1	26,2
6 <sup>th</sup>	-	-	1170,0	11,6

In figure 3 the mode shapes of the leaf spring CFRP1 at the two different temperatures are shown. It is noticeable that the cooled test body generated two closed coupled modes at the beginning. Both mode shapes refer to only one node. The second mode shape with two nodes (2<sup>nd</sup> at (a) and 3<sup>rd</sup> at (b)) has almost the same frequency, but a different damping value. This experiment shows that lower temperatures lead to higher damping.

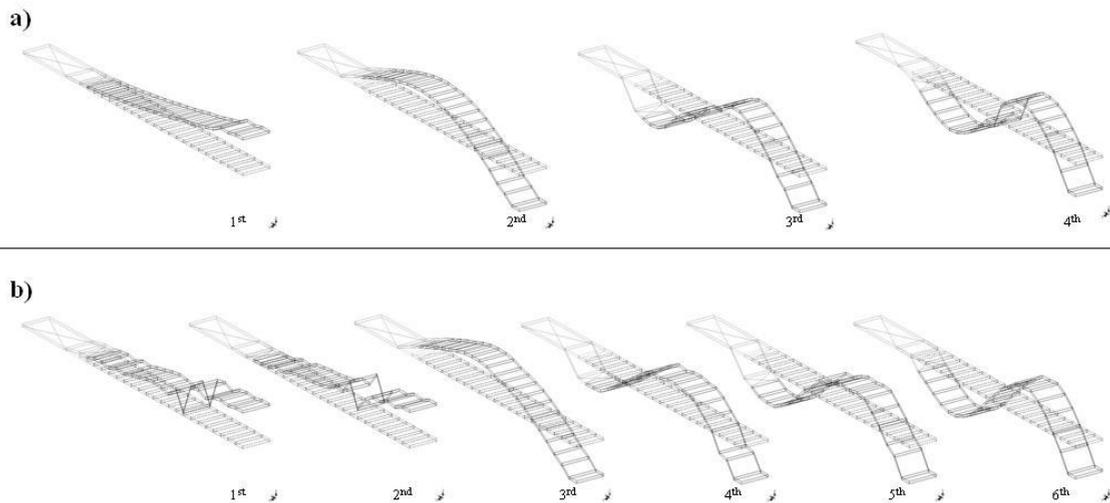


Fig. 3. (a) CFRP1 at 23°C; (b) CFRP1 at -8°C.

The correlation between the estimated damping ratio of each vibration mode and the resonance frequency of the CFRP1 leaf spring at 23°C and -8°C is shown in figure. 4. One can see that the modal damping ratio of each vibration mode is drifting apart and converge again at mode number six of the cooled down leaf spring. However, the modal damping ratio of the fourth mode of the cooled down leaf spring is noticeably higher than those of the leaf spring at 23°C. In figure 4 the associated relationship between the modal damping ratio and the resonance frequency of the experiment is given.

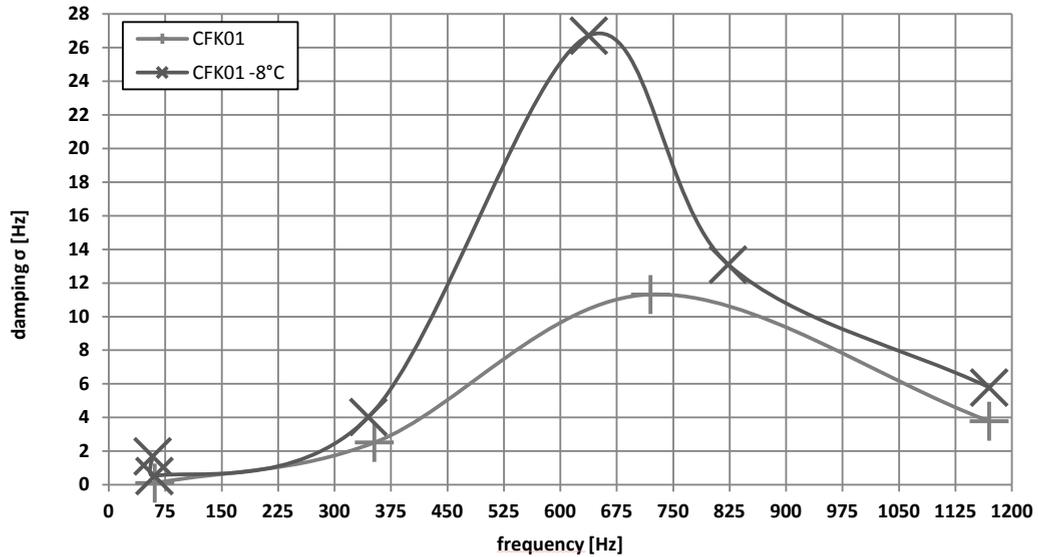


Fig. 4. Relationship between the modal damping ratio and the resonance frequency of CFRP1 at 23°C and CFRP1 at -8°C.

The impact testing has been applied because of the influence of the shaker testing on the damping values. One main advantage is that there is no physical connection between the excitation source and the cantilevered leaf spring. The impact test avoids the problem of interaction between them. Thus, it is possible to measure accurate damping quantities of the investigated bodies, but requires an unweighted signal to avoid distortion of damping by windowing the signal [8].

In table 4 the results of the impact testing can be seen and hereinafter the mode shapes of the presented eigenfrequencies are displayed in figure 5.

Table 4. Results of the impact test.

mode shape	Steel		CFRP1		CFRP2		CFRP3	
	frequency [Hz]	damping [Hz]						
1 <sup>st</sup>	21,8	0,26	57,8	0,97	39,3	0,105	35,1	0,139
2 <sup>nd</sup>	136	0,186	355,4	1,67	243,5	0,86	219,3	1,04
3 <sup>rd</sup>	380	0,76	974,5	3,9	486,8	1,44	611,2	2,26
4 <sup>th</sup>	740,8	1,55			677,3	2,28	1184	3,92
5 <sup>th</sup>	1220	2,52						

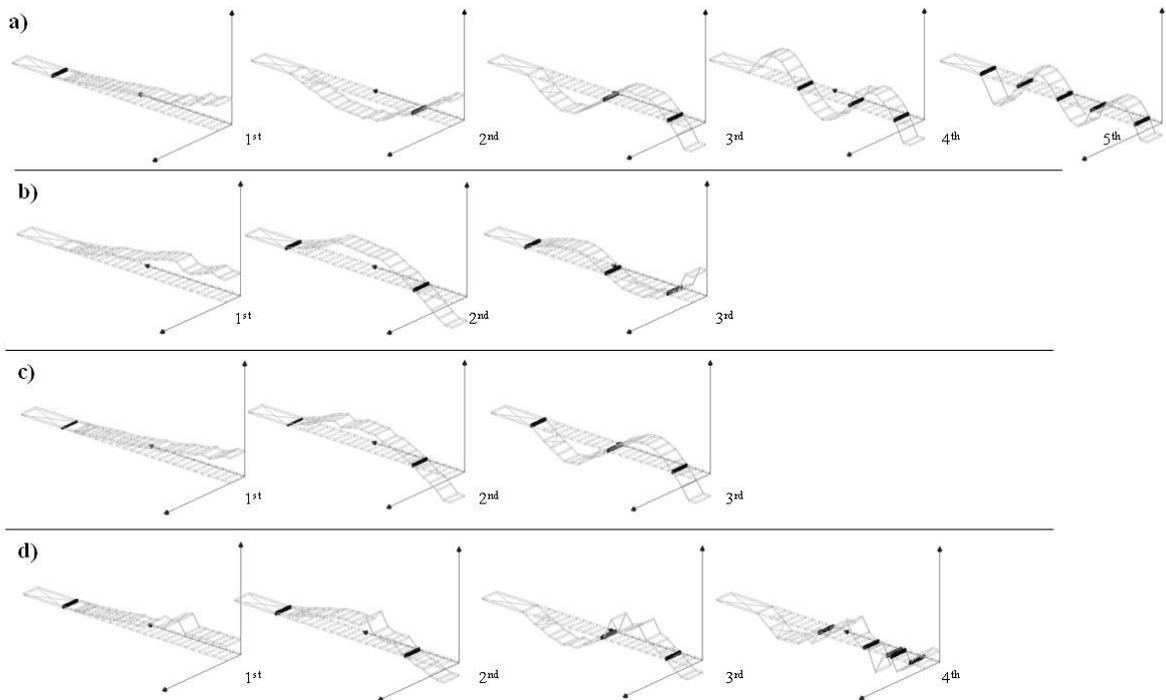


Fig. 5. Mode shapes at resonance frequencies of impact tests; (a) Steel; (b) CFRP1; (c) CFRP2; (d) CFRP3.

It's obvious that the form of the mode shapes is uniform beyond all tested leaf springs. There is no influence of an additional connection of a shaker. The nodes are rising with increasing order.

## 6. Analytical calculation of bending eigenfrequencies of a cantilever

In order to carry out verification of the measured data the Euler – Bernoulli beam theory is applied and its suitability for the leaf springs is verified. The bending vibration in the XZ-plane of a cantilevered leaf spring is defined by the partial differential equation (1). The boundary conditions for the support of a cantilevered beam and the derivation of the partial differential equation (2) are referred to [9].

$$c^2 \frac{\partial^4 \omega}{\partial x^4} + \frac{\partial^2 \omega}{\partial t^2} = 0 \quad (1)$$

$$c^2 = \frac{EJ}{\rho F} \quad (2)$$

In consideration of the parameters of table 1, the calculated eigenfrequencies for a cantilevered beam are given in table 5.

Table 5. Comparison of the measured and the analytically calculated eigenfrequencies.

mode shape	Steel		CFRP1		CFRP2		CFRP3	
	measured frequency [Hz]	analytical solution [Hz]						
1 <sup>st</sup>	21,8	21,1	57,8	55,7	39,3	37,1	35,1	34,1
2 <sup>nd</sup>	136	132,4	355,4	349,4	243,5	232,7	219,3	213,8
3 <sup>rd</sup>	380	370,7	974,5	978,3	677,3	651,5	611,2	598,7
4 <sup>th</sup>	740,8	726,4		1917			1184	1173,2
5 <sup>th</sup>	1220	1200,6		3168,9				

The comparison in figure 6 shows the correlation between the calculated and the measured eigenfrequencies.

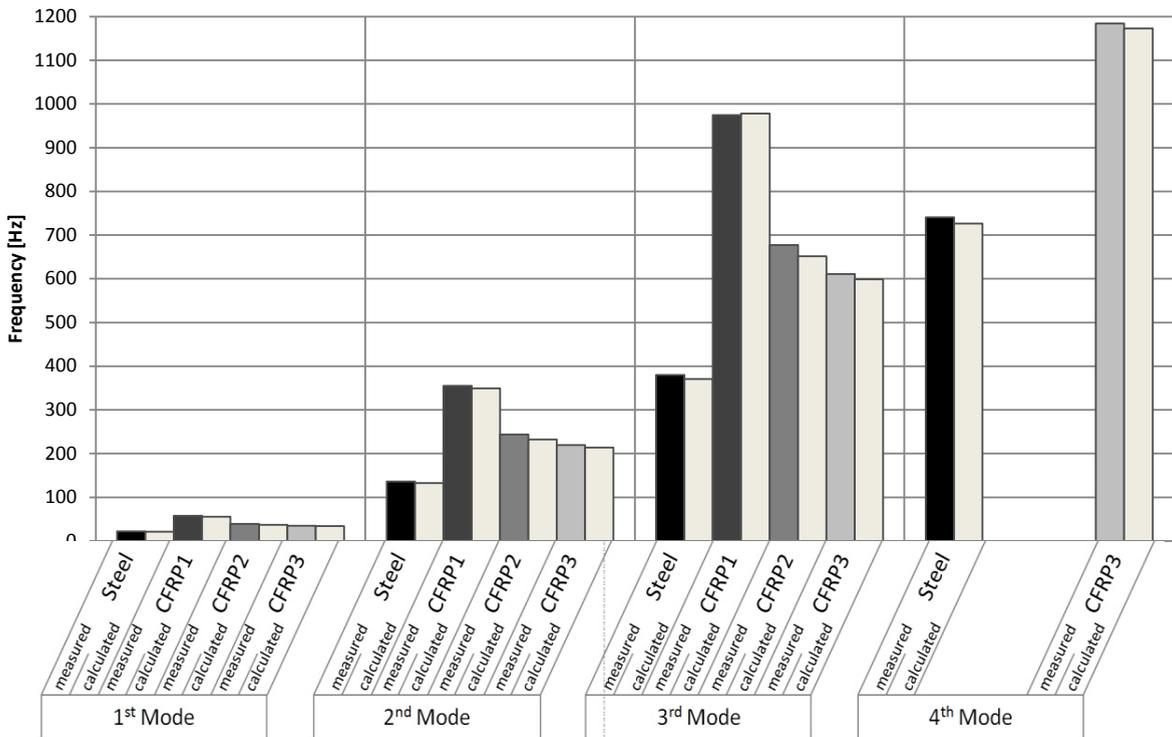


Fig. 6. Comparison between analytical calculation and measuring.

**Conclusion**

In figure 7 an arrangement of both measurements – excitation by shaker and impact excitation – is given by the modal damping ratio correlating to the resonance frequency of the test bodies. There are noticeable differences between the results of the shaker excitation and the impact testing. All dashed lines are results of the impact excitation. This occurs mainly due to the fact that there is no physical connection between the excitation and the structure when performing the impact test. Thus, the impact test avoids the problem of interaction between the structure and the excitation unit [8]. This determines the different results between the measurement methods. The point of coupling of the shaker excited leaf spring makes exactly the same movement as the shaker itself. Thus, the

best solution to determine the material properties and the dynamic characteristics – particularly for light weight structures like the leaf spring bodies – is an excitement by an impulse.

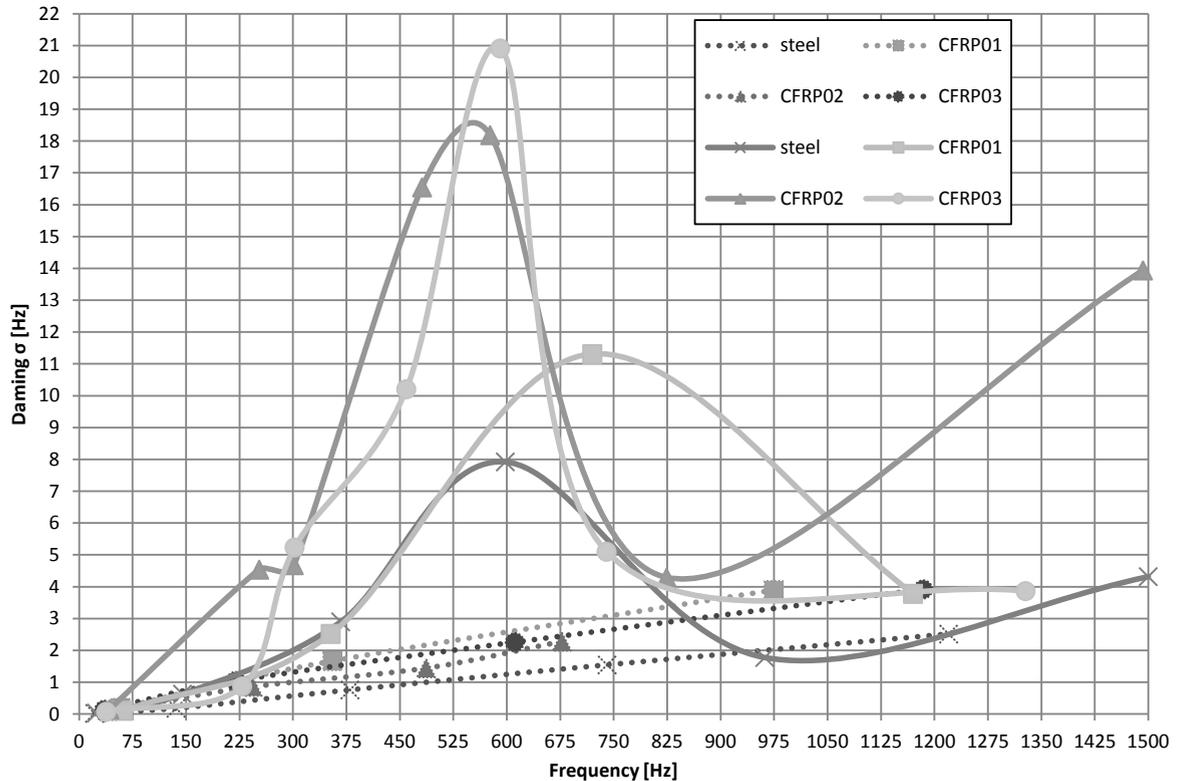


Fig. 7. Comparison of shaker and impact testing.

Another result is the confirmation of the dependence between dynamic behavior and temperature. At lower temperatures the modal damping increases.

The analytical calculation shows that the CFRP springs can be calculated like a homogeneous body with constant Young's modulus providing a good approximation. Optimization in the construction of simple composite structures can be done by using the Euler – Bernoulli beam theory.

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