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# Data-Driven Method of Fault Detection in Technical Systems

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**Abstract**

The paper is devoted to the problem of fault detection in technical systems described by nonlinear dynamical models containing non-smooth nonlinearities. So-called “model-free” or “data-driven” method is used to solve the problem. The feature of this method is that parameters of the system under consideration may be unknown. The algebra of functions is used to solve the problem under consideration.


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## 1. Introduction

The problem of fault diagnosis in technical systems was extensively investigated for the past 25 years; see, e.g., the papers [6, 7, 8, 15-17], the books [2, 3, 9-11]. Many problems have been studied and solved: different methods of residual generation and relationships among them, robustness, and adaptive threshold test, HB□B approach, fuzzy logic, descriptor systems, different classes of nonlinear systems, and so on. Many practical examples were considered; see, for example, books devoted to industrial and mechatronic systems [3, 10].There exists a promising method of fault diagnosis in technical systems described by linear and nonlinear models known as “model-free” or “data-driven” method (see, for example, the papers [1, 4, 5, 13, 14]). The feature of this method is that parameters of the system under consideration may be unknown. Therefore, this method may be named as the non-parametric one as well.

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To make a decision about faults, the method developed in [1, 4, 13, 14] is based on computation of the kernel of the matrix generated by measurements and is of high computational complexity. In addition, this method does not take into consideration different disturbances which may result in erroneous decisions about faults. Besides, the above papers consider the systems described by linear or polynomial models.

In contrast to this, the present paper suggests the way developing the model-free method for technical systems described by nonlinear dynamic models with non-smooth nonlinearities. It is well-known that actual technical systems contain such typical non-smooth nonlinearities as saturation, Coulomb friction, backlash and hysteresis. Therefore, the methods of fault diagnosis in the systems with such types of nonlinearities are necessary for practical applications. The method suggested in the paper is applicable not only for such types but for other ones. Besides, another method for decision making is suggested in this paper. In some cases it allows to decrease complexity of calculations and to take into account the disturbances in comparison with the method considered in [1, 4, 13, 14].

Consider a class of technical systems described by nonlinear models

$$x(t+1) = f(x(t), u(t)), \qquad y(t) = h(x(t)), \tag{1}$$

where $x \in X \subseteq R^n$, $u \in U \subseteq R^m$, $y \in Y \subseteq R^l$ are the vectors of state, control and output; $f$ и $h$ are nonlinear functions, the function $f$ may be non-smooth. To implement the data-driven method, make a coordinates transformations

$$x_{*i}(t) = \varphi^{(i)}(x(t)), \quad i = 1,2,..,n, \qquad y_*(t) = \psi(y(t)), \tag{2}$$

based on some functions $\varphi^{(1)}$, $\varphi^{(2)}$,…, $\varphi^{(n)}$, and $\psi$ such that in new coordinates the system is described by the model without feedbacks:

$$\begin{aligned} &x_{*1}(t+1) = f_*^{(1)}(y(t), u(t)), \\ &x_{*i}(t+1) = f_*^{(i)}(y(t), x_{*i-1}(t), ..., x_{*1}(t), u(t))), \qquad i = 2,3,...,n, \\ &y_*(t) = h_*(x_*^{(n)}(t)) \end{aligned} \tag{3}$$

for some functions $f_*^{(1)}$, $f_*^{(2)}$,…, $f_*^{(n)}$, and $h_*$. To find out a possibility to make such a transformation, special mathematical techniques so-called the algebra of functions will be used. We recall briefly the definitions and concepts to be used in this paper, see also [12].

## 2. Algebra of functions

Vector functions are elements of this algebra, which includes some binary relations, operations and operator.

1. Partial preordering relation ≤: for any functions $\alpha : X \to S$ and $\beta : X \to W$ denote $\alpha \le \beta$ if $\delta\alpha = \beta$ for some function $\delta : S \to W$, i.e. $\delta(\alpha(x)) = \beta(x)$ for all $x \in X$ where $S$ and $W$ are some sets.

2. Equivalence relation $\cong$: if $\alpha \le \beta$ and $\beta \le \alpha$, then $\alpha$ and $\beta$ are equivalent denoted $\alpha \cong \beta$.

3. Operations × and ⊕:

$$\alpha \times \beta = \max(\delta \mid \delta \le \alpha,\ \delta \le \beta),$$
$$\alpha \oplus \beta = \min(\delta \mid \alpha \le \delta,\ \beta \le \delta).$$

Clearly, the operation × gives a maximal bottom of the functions $\alpha$ and $\beta$ while the operation ⊕ gives their minimal top.

4. Binary relation $\Delta$: $(\alpha, \beta) \in \Delta$, if equality $\beta(f(x,u)) = \delta(\alpha(x), u)$ holds for some function $\delta : S \times U \to W$ and

all $(x,u) \in X \times U$.

5. Operator **m**: $\mathbf{m}(\alpha)$ is a function satisfying the conditions

$$(\alpha, \mathbf{m}(\alpha)) \in \Delta, \quad (\alpha,\beta) \in \Delta \;\Rightarrow\; \mathbf{m}(\alpha) \le \beta .$$

The relations ≤ and Δ, operations and operator have the following main properties:

1. $(\alpha \times \beta)\delta = \alpha\delta \times \beta\delta$;
2. $(\alpha,\beta) \in \Delta \;\Leftrightarrow\; \mathbf{m}(\alpha) \le \beta$;
3. if $\alpha \le \beta$, then $\mathbf{m}(\alpha) \le \mathbf{m}(\beta)$.

**3. Main theoretical result**

Theorem below stands the condition for the coordinate transformation allowing to obtain the model (3) without feedback.

**Theorem.** System (1) can be transformed into the form (3) if and only if

$$\mathbf{m}(h \times (\mathbf{m}(h \times \ldots \times \mathbf{m}(h)\ldots)) \oplus h \neq const \tag{4}$$

where the operator $\mathbf{m}$ is used $n$ times.

**Proof. Necessity.** Assume that $\varphi^{(1)}$, $\varphi^{(2)}$,…, $\varphi^{(n)}$, and $\psi$ are functions with necessary property, that is equations (3) are valid for the transformed coordinates (2). Replace the variable $x_{*1}(t+1)$ in the first equation in (3) by $\varphi^{(1)}(x(t+1))$ and $y(t)$ by $h(x(t))$:

$$\varphi^{(1)}(x(t+1)) = f_*^{(1)}(h(x(t)), u(t)) .$$

By the definition of relation $\Delta$, this equality is equivalent to the inclusion $(h, \varphi^{(1)}) \in \Delta$ and the inequality $\varphi^{(1)} \ge \mathbf{m}(h)$. Analogously, the second equation in (3) gives the inequality

$$\varphi^{(i)} \ge \mathbf{m}(h \times \varphi^{(i-1)} \times \ldots \times \varphi^{(1)}), \quad i = 2,3,\ldots,n .$$

By the properties of operation × and operator $\mathbf{m}$, one obtains $\varphi^{(2)} \ge \mathbf{m}(h \times \varphi^{(1)}) \ge \mathbf{m}(h \times \mathbf{m}(h))$ for $i = 2$ and $\varphi^{(3)} \ge \mathbf{m}(h \times \varphi^{(2)} \times \varphi^{(1)}) \ge \mathbf{m}(h \times \mathbf{m}(h) \times \mathbf{m}(h \times \mathbf{m}(h)))$ for $i = 3$.

Since $h \ge h \times \mathbf{m}(h)$, then $\mathbf{m}(h) \ge \mathbf{m}(h \times \mathbf{m}(h))$ and $\mathbf{m}(h) \times \mathbf{m}(h \times \mathbf{m}(h)) \cong \mathbf{m}(h \times \mathbf{m}(h))$, that gives

$$\varphi^{(3)} \ge \mathbf{m}(h \times \mathbf{m}(h \times \mathbf{m}(h))) .$$

By analogy one obtains

$$\varphi^{(n)} \ge \mathbf{m}(h \times (\mathbf{m}(h \times \ldots \times \mathbf{m}(h)\ldots)) \tag{5}$$

(the operator $\mathbf{m}$ is used $n$ times).

The equality $y_*(t) = \psi(y(t))$ with $y_*(t) = h_*(x_{*n}(t))$ and $y(t) = h(x(t))$ can be rewritten in the form

$$h_*(\varphi^{(n)}(x(t))) = \psi(h(x(t))) .$$

Taking into account the relation (5) for the function $\varphi^{(n)}$, one obtains

$$h_*(\mathbf{m}(h\times(\mathbf{m}(h\times\ldots\times\mathbf{m}(h)\ldots)))\geq\psi h\geq h\,. \tag{6}$$

Since

$$h_*(\mathbf{m}(h\times(\mathbf{m}(h\times\ldots\times\mathbf{m}(h)\ldots)))\geq\mathbf{m}(h\times(\mathbf{m}(h\times\ldots\times\mathbf{m}(h)\ldots)) \tag{7}$$

by the definition of relation $\geq$, it follows from (6) and (7) and the property of operation $\oplus$ that

$$h_*(\mathbf{m}(h\times(\mathbf{m}(h\times\ldots\times\mathbf{m}(h)\ldots)))\geq\mathbf{m}(h\times(\mathbf{m}(h\times\ldots\times\mathbf{m}(h)\ldots))\oplus h\,.$$

Finally, since $h_*(\mathbf{m}(h\times(\mathbf{m}(h\times\ldots\times\mathbf{m}(h)\ldots)))\neq const$, then $\mathbf{m}(h\times(\mathbf{m}(h\times\ldots\times\mathbf{m}(h)\ldots))\oplus h\neq const$, that is (4) is true.

**Sufficiency.** Different functions $\varphi^{(1)}$, $\varphi^{(2)}$,…, $\varphi^{(n)}$ can be obtained based on the condition (4), therefore one can obtain different feedback-free models. Consider simpler model than the one described by (3):

$$x_{*1}(t+1)=f_*^{(1)}(y(t),u(t)),$$

$$x_{*i}(t+1)=f_*^{(i)}(y(t),x_{*i-1}(t),u(t))),\qquad i=2,3,..,n\,, \tag{8}$$

$$y_*(t)=h_*(x_{*n}(t))\,. \tag{9}$$

Set $\varphi^{(1)}=\mathbf{m}(h)$, $x_{*1}=\varphi^{(1)}(x)$, $\varphi^{(i)}=\mathbf{m}(h\times\varphi^{(i-1)})$, $x_{*i}=\varphi^{(i)}(x)$, $i=2,3,..,n$. By the definition of operator $\mathbf{m}$, one has $(\alpha,\mathbf{m}(\alpha))\in\Delta$ for arbitrary function $\alpha$, therefore $(h,\mathbf{m}(h))\in\Delta$ and $(h\times\varphi^{(i-1)},\mathbf{m}(h\times\varphi^{(i-1)}))\in\Delta$, that gives after appropriate replacements $(h,\varphi^{(1)})\in\Delta$ and

$$(h\times\varphi^{(i-1)},\varphi^{(i)})\in\Delta\,,\ i=2,3,..,n\,.$$

This means by the definition of relation $\Delta$ that (8) is true for some functions $f_*^{(1)}$, $f_*^{(2)}$,…, $f_*^{(n)}$. By the definition of functions $\varphi^{(1)}$, $\varphi^{(2)}$,…, $\varphi^{(n)}$, one obtains $\varphi^{(n)}=\mathbf{m}(h\times(\mathbf{m}(h\times\ldots\times\mathbf{m}(h)\ldots))$ (the operator $\mathbf{m}$ is used $n$ times). It follows from this relation and the condition (4) that there exit the functions $h_*$ and $\psi$ such that $h_*(\varphi^{(n)}(x(t)))=\psi(h(x(t)))$, or $y_*(t)=h_*(x_{*n}(t))=\psi(y(t))$. The theorem has been proved.

It can be shown that theorem is true for continuous-time system as well.

Notice that the output $y_*$ is found in the form $y_*(t)=h_*(x_{*1}(t),x_{*2}(t),\ldots,x_{*n}(t))$. To simplify the transformed system, it is recommended to choose the function $\varphi^{(i)}$, $i=2,3,..,n$, with minimal number of components such that $\varphi^{(1)}\times\varphi^{(2)}\times\ldots\times\varphi^{(i)}\cong\mathbf{m}(h\times(\mathbf{m}(h\times\ldots\times\mathbf{m}(h)\ldots))$, where $\varphi^{(1)}=\mathbf{m}(h)$ (the operator $\mathbf{m}$ is used $i$ times). Otherwise, if we set $\varphi^{(i)}=\mathbf{m}(h\times(\mathbf{m}(h\times\ldots\times\mathbf{m}(h)\ldots))$, the dimension of the transformed system increases. Note that such a choice of the functions $\varphi^{(i)}$, $i=2,3,..,n$, yields a presentation of the initial system in the form (3).

Thus, if the condition (4) holds, the initial system can be transformed into the form (3). To apply the main ideas of the model-free method, transform equations (3) into the single equation using several time shifts and substitutions of $f_*^{(i)}(y(t),x_{*i-1}(t),u(t))$ from (8) for $x_{*i-1}(t+1)$, $i=1,2,\ldots,n-1$.

## 4. Computational relations

Based on time shifts and substitutions, one can transform equations (3) into the single equation as follows:

$$x_{*2}(t+2)=f_*^{(2)}(y(t+1),x_{*1}(t+1),u(t+1))=f_*^{(2)}(y(t+1),f_*^{(1)}(y(t),u(t)),u(t+1))\,,$$

$$x_{*3}(t+3)=f_*^{(3)}(y(t+2),f_*^{(2)}(y(t+1),f_*^{(1)}(y(t),u(t)),u(t+1))),u(t+2))\,,$$

$$\dots$$

$$x_{*N}(t+n) = F_*(y(t+n-1),...,y(t),u(t+n-1),u(t))$$

for some function $F_*$. As a result, the expression (9) takes the form

$$y_*(t+n) = F_{**}(y(t+n-1),...,y(t),u(t+n-1),u(t))), \tag{10}$$

where $F_{**} = h_* F_*$.

Assume for simplicity that the functions $f$ and $h$ are polynomials and the function $f$ contains the unknown parameters $\gamma$. Assuming that $\varphi^{(1)}$, $\varphi^{(2)}$,…, $\varphi^{(n)}$ and $\psi$ are polynomials as well, one obtains that $f_*^{(1)}$, $f_*^{(2)}$,…, $f_*^{(n)}$ and $h_*$ are polynomials also. This means that the right-hand side of (10) can be written in the form

$$F_{**}(y(t+n-1),...,y(t),u(t+n-1),u(t))) = \begin{pmatrix}\Gamma_1(\gamma) & \Gamma_2(\gamma) & \dots & \Gamma_v(\gamma)\end{pmatrix}\begin{pmatrix} P_1(y,u,t+n-1,...,t) \\ P_2(y,u,t+n-1,...,t) \\ \vdots \\ P_v(y,u,t+n-1,...,t)\end{pmatrix}, \tag{11}$$

where $\Gamma_1(\gamma)$, $\Gamma_2(\gamma)$,…, $\Gamma_v(\gamma)$ are some expressions containing the parameter $\gamma$, $P_1(y,u,t+n-1,...,t)$, $P_2(y,u,t+n-1,...,t)$,…, $P_v(y,u,t+n-1,...,t)$ are polynomials containing vectors of control and output with arguments from $t+n-1$ till $t$. Based on (11), write down the expression for $y_*(t+n)$ for several time instants:

$$\begin{gathered} Y_*(k) = \begin{pmatrix} y_*(t+n+k) & y_*(t+n+k-1) & \dots & y_*(t+n)\end{pmatrix} = (\Gamma_1(\gamma) \quad \Gamma_2(\gamma) \quad \dots \quad \Gamma_v(\gamma)) \cdot \\ \cdot \begin{pmatrix} P_1(y,u,t+n+k-1,...,t+k) & P_1(y,u,t+n+k-2,...,t+k-1) & \dots & P_1(y,u,t+n-1,...,t) \\ P_2(y,u,t+n+k-1,...,t+k) & P_2(y,u,t+n+k-2,...,t+k-1) & \dots & P_2(y,u,t+n-1,...,t) \\ \vdots & \vdots & \ddots & \vdots \\ P_v(y,u,t+n+k-1,...,t+k) & P_v(y,u,t+n+k-2,...,t+k-1) & \dots & P_v(y,u,t+n-1,...,t) \end{pmatrix} = \\ (\Gamma_1(\gamma) \quad \Gamma_2(\gamma) \quad \dots \quad \Gamma_v(\gamma)) V_n. \end{gathered} \tag{12}$$

The expression (12) is a starting point to make a decision about faults in the initial system (1). Note that the matrices $Y_*(k)$ and $V_n$ in (12) contain the known measurements while the matrix $(\Gamma_1(\gamma) \quad \Gamma_2(\gamma) \quad \dots \quad \Gamma_v(\gamma))$ contains the unknown parameter $\gamma$.

## 5. Decision making

Consider rows of the matrix $V_n$ in (12) as a set of the basis vectors of some hyperplane denoted by $\mathbf{L}(V_n)$. If the vector $Y_*(n)$ belongs to this hyperplane, one concludes that the system is healthy. In the presence of disturbances, this vector does not need to belong to the hyperplane $\mathbf{L}(V_n)$ even in the faulty-free case; the value of distance between the vector $Y_*(n)$ and the hyperplane $\mathbf{L}(V_n)$ can be used for decision making on the basis of some threshold.

To calculate the distance between the vector $Y_*(n)$ and the hyperplane $\mathbf{L}(V_n)$, the following decomposition of the matrix $V_n$ is used:

$$V_n = A \cdot \Sigma \cdot B, \tag{13}$$

where $A$ and $B$ are nonsingular matrices, $\Sigma = \begin{pmatrix} I & 0 \end{pmatrix}$. Such a decomposition can be obtained on the basis of singular value decomposition of the matrix $V_n$ suggested for the purpose of diagnosis in [6].

Suppose that $Y_*(n) \in \mathbf{L}(V_n)$, i.e. the vector $Y_*(n)$ is a linear combination of the matrix $V_n$ rows. Therefore the equation $Y_*(n) = CV_n$, or $Y_*(n) = C \cdot A \cdot \Sigma \cdot B$ holds for some matrix $C$. Compute the vector $Y = Y_*(n)B^{-1}$ and rewrite the previous expression in the form $Y = C \cdot A \cdot \Sigma$. It follows from this expression that the vector $Y$ is a linear combination of the matrix $\Sigma = \begin{pmatrix} I & 0 \end{pmatrix}$ rows. This matrix allows concluding that the last element of the vector $Y = Y_*(n)B^{-1}$ is equal to zero in the faulty-free case. Denoting this element by $Y_L$, one can consider the equality $Y_L = 0$ as a parity relation. Therefore, to generate a residual, the expression $r(n) = Y_L$ can be used.

It follows from the structure of the matrix $\Sigma = \begin{pmatrix} I & 0 \end{pmatrix}$ that for the arbitrary vector $Y_*(n)$ the row $C'$ exists such that the difference $Y_*(n)B^{-1} - C' \cdot A \cdot \Sigma$ is a vector with all zero components but the last one. This means that the value $r(n) = Y_L$ can be considered as a distance between the vector $Y_*(n)$ and the hyperplane $\mathbf{L}(V_n)$.

The decision making rule is as follows: if $r(n) = 0$, the system is health, otherwise the fault has occurred. In the presence of disturbances, the residual $r(n)$ should be compared with the threshold has to be determined. Notice that according to (12), this rule is not based on the parameters of the system, they may be unknown.

## Conclusion

The paper is devoted to the problem of fault detection in technical systems described by nonlinear models with non-smooth nonlinearities. So-called model-free method to solve this problem is considered. The feature of this method is that parameters of the system under consideration may be unknown. The suggested solution is based on singular value decomposition and allows to decrease complexity of calculations in comparison with the method considered in [1, 4, 13, 14] in some cases.

The future plan of researches is: (1) comparison between the algebraic and geometric approaches; (2) development of procedure to design the threshold; (3) establishment of conditions under which fault isolation is possible.

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