24th DAAAM International Symposium on Intelligent Manufacturing and Automation, 2013

Multi-Criteria Optimization of Insulation Options for Warmth of Buildings to Increase Energy Efficiency

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Abstract

Increasing energy efficiency and reducing CO₂ are among the main drivers of modern scientific research. One of the solutions to increase energy efficiency is the selection of the insulation of the walls on buildings. The aim of this study is to optimize such insulation taking into account several criteria.

The paper analyzes the state of the buildings in Sarajevo, precisely in Alipasino polje, which the company KJKP „Toplane-Sarajevo“ supplies with thermal energy. Several options for building warmth are considered and evaluated by selected criteria, after that multi-criteria optimization method VIKOR was applied to rank the options and select the best one. Optimization on the basis of relevant criteria is done before making investment project for warmth buildings, to provide input parameters for designing and decision-making processes.

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Selection and peer-review under responsibility of DAAAM International Vienna

Keywords: energy efficiency; warmth building; multi-criteria optimization; insulation material; vikor method

1. Introduction

To maximize energy efficiency by installing the insulation on buildings, it’s recommended to use multi-criteria optimization [1]. It should be noted that at present a purely economic analysis is not sufficient and that concerns for depletion of fossil fuels are to be taken into the account [2]. Therefore, the analysis becomes more complicated when needed to get energy efficiency and at the same time to reduce both costs and CO₂ emissions. The reason is that the minimization of costs, increase of energy efficiency and CO₂ reduction are opposed objectives. Multi-

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criteria optimization is used to resolve these problems by combining the desired criteria [3]. Multi-criteria optimization provides a solution when the objective functions are opposing. It may find set of solutions for each criteria separately, and then, from that set, provides an optimal solution to the overall problem.

Such solution depends on changes in coefficients showing the criteria importance, or on criteria exclusion [4]. In order to improve the operation of district-heating systems, it is necessary for the energy companies to have reliable computerized optimization routines implemented in their organizations [5]. In this context, decision support systems and tools are needed for providing decision makers with specific functionalities for integrating environmental and socio-economic factors, for comparing and selecting alternatives. Ranking should be based on understandable and measurable criteria, enabling thus decision makers to make the informed decisions with optimal results.

The aim of this study is to present the importance of energy management in buildings and installing the thermal insulation on buildings respecting existing regulations and standards. This paper presents the research primarily on saving energy, reducing CO₂ emissions and reducing the cost of investment, but considering also other criteria that are important to the district heating system „Toplane-Sarajevo“ [6]. On the basis of the selected criteria, the optimal solution of the type of thermal insulation of walls on buildings is obtained. That will increase the energy efficiency in the thermal energy distribution system and reduce CO₂ emissions.

2. Problem description

Existing buildings in Alipasino polje consume enormous amounts of energy for heating. That contributes to the environment pollution, and most of them were built by the standards of 50 years ago. With an economic, environment and energy savings, the comfort and quality of life in buildings would be improved with extended life of buildings resulting by greater energy efficiency. Also, low heat transfer coefficient is necessary in order to decrease thermal losses in the buildings. Thermal insulation is carried out, as a rule, by adding thermal insulation layer on the outer side of walls and only in exceptional cases inside of walls. Installation of insulation on the inside of wall is unfavourable to construction-physical point of view, and often is more expensive due to the need of additional problem solving of water vapor diffusion, more stringent requirements in terms of security against fire, loss of usable space, etc. The use of thermal insulation on the inside wall is physically weaker, although improved insulation value of the wall is achieved, significantly changing the heat flux in the wall and the main load-bearing wall becomes colder, resulting in the need for performing a vapor barrier to prevent the formation of condensate [7]. Because of these shortcomings, it is much better to use an external insulation of buildings. On the market there are a wide range of materials for outside insulation, so it’s necessary to optimize the desired criteria in order to select the best alternative of insulation. It’s necessary to know that in the winter building must retain as much heat energy as possible, and in the summer not to accumulate energy from the outside. The choice of materials must satisfy all regulations and calculations.

3. Solution and alternatives

Based on local BiH market research, available materials for insulation of walls on buildings are styrofoam, mineral wool (stone wool and glass wool), pluto panels, polyester, polyurethane, perlite and wood wool.

3.1. Styrofoam

Styrofoam has good insulating properties, relatively low cost and easy mount. Therefore, today is one of the most popular insulation materials. Styrofoam is resistant to temperatures over 80°C.

3.2. Mineral wool - stone wool and glass wool

Stone wool has a high resistance to fire, it is vapor-permeable and partially waterproof. It is resistant to aging and decay, and has a high compressive strength. Stone wool is obtained from the mineral stone, dolomite, basalt and diabase with the addition of coke.
Glass wool is among better materials for insulation. The main raw material used to produce glass wool is quartz sand with the addition of recycled glass. Stone wool has a lower density than rock wool and lower compressive strength, while the insulating properties are approximated to rock wool.

![Fig. 1. (a) Styrofoam; (b) Stone wool; (c) Glass wool.](image)

3.3. Pluto panels

Pluto is a very good thermal insulator, and it is obtained from the bark of the cork oak tree. Cork boards are produced by grinding the bark or by expanding particles floating in autoclaves and then cutting the blocks in expanded cork boards of various lengths. Plates can be used as a decorative surface, since looking nice.

3.4. Polyester

Polyester has insulating properties similar to mineral wool (rock and glass wool). Polyester is used for the production of various types of clothing and like by virtue of its characteristic.

![Fig. 2. (a) Pluto; (b) Polyester.](image)

3.5. Polyurethane

Polyurethane is widely used because it has a lot of good insulating properties. Polyurethane has a good resistance to humidity and temperature changes. One of the drawbacks is that the polyurethane is more expensive than styrofoam and fiberglass. Polyurethane is resistant to temperatures up to 250°C briefly, so that the panel of polyurethane foam is suitable as a substrate. Polyurethane foam can spurt on the surface or in a cavity.

3.6. Perlite

Perlite is a volcanic aluminium-silicate rock, which is mechanically crushed and briefly heated to a temperature of 1000 °C. In such way, the water contained in the rock turns into water vapor and inflates material and increase the
volume of 15 to 20 times. This procedure gives expanded perlite. Individual grains are composed of cells that are responsible for the thermal insulation properties. Perlite is used mainly as insulating material for backfilling. Expanded perlite is not flammable but is sensitive to moisture.

3.7. Wood wool

Wood wool is produced so that the fibers are combined with cement. Panels are lightweight because they contain cavities. For thermal insulation a mixture of wood particles and a binder can be used.

![Fig. 3. (a) Polyurethane; (b) Expanded perlite; (c) Wood wool.](image)

4. Criteria selection and valuating

The decision of the material for insulation is based on the selected criteria for which the alternatives are valued, ranked and made a final selection. The model used is based on the following selected criteria:

a. Costs of insulation for $U=0.40\text{W/m}^2\text{K}$ in €, criteria to be minimized
b. The density of the material in kg/m$^3$, criteria to be maximized
c. Specific heat in J/kgK, criteria to be minimized
d. Thermal conductivity W/mK, criteria to be minimized
e. Water vapor diffusion resistance factor, criteria to be maximized
f. Emission of CO$_2$ in kgCO$_2$/kg, criteria to be minimized

The costs of insulation is determined by the actual pricing of mentioned materials in the market. The values of other criteria are known for each listed material [8].

Costs of insulation per each of the alternatives, presented in this paper, are obtained in local market and they correspond to integrating and appropriate thickness of material to obtain a heat transfer coefficient $U=0.40\text{W/m}^2\text{K}$, which corresponds to the new European standards. Specifically, for styrofoam the thickness is 10 cm. Other materials are compared to insulation properties and cost of styrofoam of 10 cm thickness. If the material is weaker insulator, it means that a larger thickness is required for the same effect, and vice versa. The density of the material describes the relationship of mass and volume of a substance. Specific heat is the energy required to raise the temperature of unit mass per unit of temperature. A higher value means a higher specific heat capacity of thermal energy storage. Thermal conductivity is the property of a material to conduct heat. Thus, lower value of thermal conductivity means a better insulator. Water vapor diffusion resistance factor is the resistance of material to the abandonment of water vapor into air, describes how water vapor penetrates through the material. If the temperature of the buildings is reducing, the steam is converted into dew and it favours to formation of moisture. The smaller value of this factor means greater permeability. Emissions of CO$_2$ represent the amount of dioxide emissions related to production of one kilogram of insulating material.

It is appropriate to assign the weighting factor to each criterion. With the consideration that the costs are usually the most important criterion, the costs will have the largest weight factor. After costs, the emission of CO$_2$ into the air is most important, and therefore it should be reduced as far as possible. Next factor which is used is the
coefficient of thermal conductivity. The following are specific heat and water vapor diffusion resistance factor, which is given an equal weighting factor. In the end remains density with smaller weighting factor. Selected (normalized, sum equals one) weighting factors are as follows:

- Costs of insulation material: 0.40 (or 40%).
- CO₂ emission: 0.20 (or 20%).
- Coefficient of thermal conductivity: 0.15 (or 15%).
- Specific heat: 0.10 (or 10%).
- Water vapor diffusion resistance factor: 0.10 (or 10%).
- Density: 0.05 (or 5%).

Results of the type of material for insulation walls of buildings is based for heat transfer coefficient \( U = 0.40 \text{ W/m}^2\text{K} \). This value of the heat transfer coefficient satisfies the European standards [10].

5. Vikor method

Making a choice decision between several alternatives of insulation materials is part of the energy efficiency management. The multicriteria optimization is a part of the multicriteria decision making process. The basic steps in this optimization are:

- Step 1: Defining the goals and methods to achieve the goals,
- Step 2: Formalizing the problem, determining and valuation of criteria,
- Step 3: Selection and use of appropriate multicriteria optimization method, and
- Step 4: Making the final decision or re-evaluation, repeating the procedure from the second step.

Numerous different methods are developed to solve this type of problem. As an example we can list methods for determining non-inferior solutions, methods with pre-expressed preferences, interactive methods where preferences are gradually determined, stochastic methods, compromise programming, etc. Solution is not unique and it directly depends on the method selected. Since the n-dimensional space is not fully ordered, most of these methods generally tend to order the space completely or partially. Methods used specifically in water sector are described in [9].

The entire process of defining problems, determining alternatives and evaluating criteria, contour constraints, optimization and making the final choice is called multicriteria decision making [11]. The term "decision" more accurately determines the whole process, since only by decision one can select a solution (or set of solutions). The multicriteria decision making with help of Vikor method was used for prioritization of insulation alternatives. A MCDA problem can be represented by a decision matrix as follows:

\[
D = \begin{bmatrix}
A_1 & A_2 & \ldots & A_n \\
C_{x1} & C_{x2} & \ldots & C_{xn} \\
x_{11} & x_{12} & \ldots & x_{1n} \\
x_{21} & x_{22} & \ldots & x_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
x_{m1} & x_{m2} & \ldots & x_{mn}
\end{bmatrix}
\]  

(1)

Here, \( A_i \) represents \( i^{th} \) alternative, \( i=1,2,\ldots,n \); \( C_{ij} \) represents the \( j^{th} \) criterion, \( j=1,2,\ldots,m \); and \( x_{ij} \) is the individual performance of an alternative. The procedures for evaluating the best solution to an MCDA problem include computing the utilities of alternatives and based on that ranking these alternatives. The alternative solution with the highest utility is considered to be the optimal solution. The following steps are involved in VIKOR method:

- Step 1: Representation of normalized decision matrix.
- Step 2: Determination of ideal and negative-ideal solution.
- Step 3: Calculation of utility measures.
- Step 4: Computation of VIKOR index.
- Step 5: Ranking the alternatives.
5.1. Representation of normalized decision matrix

The normalized decision matrix can be expressed as follows:

\[ F = [f_{ij}]_{mxn} \]  

Here, \( x_{ij} \) is the performance of alternative \( A_i \) with respect to the \( j \)th criterion, and \( f_{ij} \) is:

\[ f_{ij} = \frac{x_{ij}}{\sqrt{\sum_{k=1}^{n} x_{ij}^2}}, i = 1,2, ... , m \]  

5.2. Determination of ideal and negative-ideal solutions

Determine the best \( A^*_i \) and the worst \( A^-_i \) values of all criterion functions, \( i = 1,2, ... , n \). If the \( i \)th function represents a benefit then \( A^*_i = \max_j A_{ij} \) and \( A^-_i = \min_j A_{ij} \), while if the \( i \)th function represents a cost \( A^*_i = \min_j A_{ij} \) and \( A^-_i = \max_j A_{ij} \), so the values are determined as follows:

\[ A^* = \{f^*_1, f^*_2, ... , f^*_n \} \]  
\[ A^- = \{f^-_1, f^-_2, ... , f^-_n \} \]

5.3. Calculation of measures \( S_i \) and \( R_i \)

The utility measures for each alternative are given as:

\[ S_i = \sum_{j=1}^{n} w_j \left( \frac{f^*_j - f^-_j}{f^*_j - f^-_j} \right) \]  
\[ R_i = \max_j \left[ w_j \left( \frac{f^*_j - f^-_j}{f^*_j - f^-_j} \right) \right] \]

where, \( S_i \) and \( R_i \), represent the utility measures and \( w_j \) is the weight of the \( j \)th criterion.

5.4. Computation of VIKOR index

The VIKOR index can be expressed as follows:

\[ Q_i = v \left[ \frac{S_i - S^*}{S^* - S^-} \right] + (1 - v) \left[ \frac{R^- - R^*}{R^- - R} \right] \]  

Here, \( Q_i \), represents the \( i \)th alternative VIKOR value, \( i = 1,2, ... , m \); \( S^* = \min_i (S_i) \); \( S^- = \max_i (S_i) \); \( R^* = \min_i (R_i) \); \( R^- = \max_i (R_i) \); and \( v \) is the weight of the maximum group utility (usually it is to be set to 0.5). The alternative having smallest VIKOR value is determined to be the best solution.

5.5. Alternative ranking

Next steps relates to ranking the alternatives, sorting by the values \( S, R \) and \( Q \) in increasing order. The results are three ranking lists. Propose as a compromise solution the alternative \( A^{(1)} \) which is the best ranked by the measure \( Q \) (minimum), if the following two conditions are satisfied:
a. Acceptable advantage \( Q(A^{(2)}) - Q(A^{(1)}) \geq DQ \) where \( DQ = \frac{1}{(J-1)} \) and \( A^{(2)} \) is the alternative with second position in the ranking list by \( Q \).

b. Acceptable stability in decision making. The alternative \( A^{(1)} \) must also be the best ranked by \( S \) or/and \( R \). This compromise solution is stable within a decision making process, which could be the strategy of maximum group utility (when \( v > 0.5 \) is needed), or “by consensus” (\( v \approx 0.5 \)), or with veto (\( v < 0.5 \)).

If one of the conditions is not satisfied, then a set of compromise solutions is proposed which consist of:

a. Alternative \( A^{(1)} \) and \( A^{(2)} \) if only condition b. is not satisfied, or

b. Alternatives \( A^{(1)}, A^{(2)}, \ldots, A^{(m)} \) if the condition a. is not satisfied. \( A^{(m)} \) is determined by the relation \( Q(A^{(m)}) - Q(A^{(1)}) < DQ \) for maximum \( n \) (the positions of these alternatives are “in closeness”).

Ranking the alternatives by the VIKOR method gives results as a compromise solution.

6. Application of MCDA

In this paper, as stated, the problem is which material to use for thermal insulation of buildings walls. The problem is addressed by using the multi-criteria analysis methods that are more flexible than strictly mathematical optimization techniques. Here the VIKOR method is used to perform ranking of alternatives to choose the material for insulation. In determining the ranking list is necessary to use the normalization of data and assign weight factors to each alternative. The reason for this is that the VIKOR method during the process of calculating uses a complex linear normalization, which is multiplied by the weight factors and thus is obtained a pessimistic and desired solution. On the basis of above written, the steps to solve this problem are as follows:

1. Determination of the minimization or maximization of each criterion (Tab. 1.).
2. Determination of MAX matrix (Tab. 2.).
3. Determination of the normalized matrix and assigning weights (Tab. 3.).
4. Determination of the matrix with the included weight factors (Tab. 4.).
5. Determination of compromise values (Tab. 5.).
6. Ranking using VIKOR method provide “success table” (Tab. 6.) [12].
7. Testing the stability (changing \( v \) value on 0.25 and 0.75) and advantage \( (DQ > Q(A^{(2)}) - Q(A^{(1)}) \)) of first ranking alternatives (Tab. 7.).

| Table 1. Determination of the minimization or maximization of each criterion. |
|---------------------------------|---------------|---------------|---------------|---------------|----------------|---------------|---------------|---------------|
|                                 | Styrofoam     | Stone wool    | Glass wool    | Cork board    | Polyester fibers | Polyurethane  | Perlite       | Wood wool     |
| Costs of insulation for U=0.4 W/m2K in € | 15.97         | 38.36         | 32.42         | 38.38         | 25.87           | 34.39         | 52.91         | 22.49         |
| Density in kg/m3               | 25.56         | 220.00        | 75.00         | 100.00        | 20.00           | 20.00         | 100.00        | 65.00         |
| Specific heat in J/kgK         | 767           | 760           | 840           | 1,800         | 1,600           | 1,500         | 1,000         | 2,100         |
| Coefficient of thermal conductivity in W/mK | 0.021         | 0.035         | 0.050         | 0.040         | 0.045           | 0.040         | 0.080         | 0.100         |
| Water vapor diffusion resistance factor | 20.45         | 1.40          | 1.00          | 22.00         | 2.00            | 5.00          | 5.00          | 2.00          |
| Emission of CO2 in kgCO2/kg    | 1.28          | 1.01          | 1.20          | 0.19          | 3.80            | 3.48          | 0.52          | 0.98          |
| min/max                         | -1            | 1             | -1            | -1            | -1              | -1            | -1            | -1            |
Table 2. Determination of MAX matrix.

<table>
<thead>
<tr>
<th></th>
<th>Styrofoam</th>
<th>Stone wool</th>
<th>Glass wool</th>
<th>Cork board</th>
<th>Polyester fibers</th>
<th>Polyurethane</th>
<th>Perlite</th>
<th>Wood wool</th>
<th>Min</th>
<th>Max</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Costs of insulation</td>
<td>-15.97</td>
<td>-38.36</td>
<td>-32.42</td>
<td>-38.38</td>
<td>-25.87</td>
<td>-34.39</td>
<td>-52.91</td>
<td>-22.49</td>
<td>-52.91</td>
<td>-15.97</td>
<td>36.94</td>
</tr>
<tr>
<td>Density</td>
<td>25.56</td>
<td>220.00</td>
<td>75.00</td>
<td>100.00</td>
<td>20.00</td>
<td>20.00</td>
<td>100.00</td>
<td>65.00</td>
<td>20.00</td>
<td>220.00</td>
<td>200.00</td>
</tr>
<tr>
<td>Specific heat</td>
<td>-767</td>
<td>-760</td>
<td>-840</td>
<td>-1,800</td>
<td>-1,600</td>
<td>-1,500</td>
<td>-1,000</td>
<td>-2,100</td>
<td>-2,100</td>
<td>-760</td>
<td>1,340</td>
</tr>
<tr>
<td>Coefficient of thermal conductivity</td>
<td>-0.02</td>
<td>-0.04</td>
<td>-0.05</td>
<td>-0.04</td>
<td>-0.05</td>
<td>-0.04</td>
<td>-0.08</td>
<td>-0.10</td>
<td>-0.10</td>
<td>-0.02</td>
<td>0.08</td>
</tr>
<tr>
<td>Water vapor diffusion resistance factor</td>
<td>20.45</td>
<td>1.40</td>
<td>1.00</td>
<td>22.00</td>
<td>2.00</td>
<td>5.00</td>
<td>5.00</td>
<td>2.00</td>
<td>1.00</td>
<td>22.00</td>
<td>21.00</td>
</tr>
<tr>
<td>Emission of CO2</td>
<td>-1.28</td>
<td>-1.01</td>
<td>-1.20</td>
<td>-0.19</td>
<td>-3.80</td>
<td>-3.48</td>
<td>-0.52</td>
<td>-0.98</td>
<td>-3.80</td>
<td>-0.19</td>
<td>3.61</td>
</tr>
</tbody>
</table>

Table 3. Determination of the normalized matrix and assigning weights.

<table>
<thead>
<tr>
<th></th>
<th>Styrofoam</th>
<th>Stone wool</th>
<th>Glass wool</th>
<th>Cork board</th>
<th>Polyester fibers</th>
<th>Polyurethane</th>
<th>Perlite</th>
<th>Wood wool</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Costs of insulation</td>
<td>0.00</td>
<td>0.61</td>
<td>0.45</td>
<td>0.61</td>
<td>0.27</td>
<td>0.50</td>
<td>1.00</td>
<td>0.18</td>
<td>40%</td>
</tr>
<tr>
<td>Density</td>
<td>0.97</td>
<td>0.00</td>
<td>0.73</td>
<td>0.60</td>
<td>1.00</td>
<td>1.00</td>
<td>0.60</td>
<td>0.78</td>
<td>5%</td>
</tr>
<tr>
<td>Specific heat</td>
<td>0.01</td>
<td>0.00</td>
<td>0.06</td>
<td>0.78</td>
<td>0.63</td>
<td>0.55</td>
<td>0.18</td>
<td>1.00</td>
<td>10%</td>
</tr>
<tr>
<td>Coefficient of thermal conductivity</td>
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<td>0.18</td>
<td>0.37</td>
<td>0.24</td>
<td>0.30</td>
<td>0.24</td>
<td>0.75</td>
<td>1.00</td>
<td>15%</td>
</tr>
<tr>
<td>Water vapor diffusion resistance factor</td>
<td>0.07</td>
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<td>1.00</td>
<td>0.00</td>
<td>0.95</td>
<td>0.81</td>
<td>0.81</td>
<td>0.95</td>
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<tr>
<td>Emission of CO2</td>
<td>0.30</td>
<td>0.23</td>
<td>0.28</td>
<td>0.00</td>
<td>1.00</td>
<td>0.91</td>
<td>0.09</td>
<td>0.22</td>
<td>20%</td>
</tr>
</tbody>
</table>

Table 4. Determination of the matrix with the included weight factors.

<table>
<thead>
<tr>
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<th>Glass wool</th>
<th>Cork board</th>
<th>Polyester fibers</th>
<th>Polyurethane</th>
<th>Perlite</th>
<th>Wood wool</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Costs of insulation</td>
<td>0.00</td>
<td>0.24</td>
<td>0.18</td>
<td>0.24</td>
<td>0.11</td>
<td>0.20</td>
<td>0.40</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>0.05</td>
<td>0.00</td>
<td>0.04</td>
<td>0.03</td>
<td>0.05</td>
<td>0.05</td>
<td>0.03</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>Specific heat</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
<td>0.08</td>
<td>0.06</td>
<td>0.06</td>
<td>0.02</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>Coefficient of thermal conductivity</td>
<td>0.00</td>
<td>0.03</td>
<td>0.06</td>
<td>0.04</td>
<td>0.05</td>
<td>0.04</td>
<td>0.11</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>Water vapor diffusion resistance factor</td>
<td>0.01</td>
<td>0.10</td>
<td>0.10</td>
<td>0.00</td>
<td>0.10</td>
<td>0.08</td>
<td>0.08</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>Emission of CO2</td>
<td>0.06</td>
<td>0.05</td>
<td>0.06</td>
<td>0.00</td>
<td>0.20</td>
<td>0.18</td>
<td>0.02</td>
<td>0.04</td>
<td></td>
</tr>
</tbody>
</table>

Table 5. Determination of compromise values.

<table>
<thead>
<tr>
<th></th>
<th>Styrofoam</th>
<th>Stone wool</th>
<th>Glass wool</th>
<th>Cork board</th>
<th>Polyester fibers</th>
<th>Polyurethane</th>
<th>Perlite</th>
<th>Wood wool</th>
<th>min (S+,R+)</th>
<th>max (S-,R-)</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>0.1171</td>
<td>0.4126</td>
<td>0.4314</td>
<td>0.3864</td>
<td>0.5608</td>
<td>0.6040</td>
<td>0.6592</td>
<td>0.4984</td>
<td>0.1171</td>
<td>0.6592</td>
<td>0.5421</td>
</tr>
<tr>
<td>R</td>
<td>0.0606</td>
<td>0.2425</td>
<td>0.1781</td>
<td>0.2427</td>
<td>0.2000</td>
<td>0.1995</td>
<td>0.4000</td>
<td>0.1500</td>
<td>0.0606</td>
<td>0.4000</td>
<td>0.3394</td>
</tr>
<tr>
<td>QS</td>
<td>0.0000</td>
<td>0.5452</td>
<td>0.5799</td>
<td>0.4968</td>
<td>0.8184</td>
<td>0.8983</td>
<td>1.0000</td>
<td>0.7034</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>QR</td>
<td>0.0000</td>
<td>0.5359</td>
<td>0.3464</td>
<td>0.5365</td>
<td>0.4108</td>
<td>0.4092</td>
<td>1.0000</td>
<td>0.2635</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6. Ranking using VIKOR method.

<table>
<thead>
<tr>
<th>v</th>
<th>Styrofoam</th>
<th>Stone wool</th>
<th>Glass wool</th>
<th>Cork board</th>
<th>Polyester fibers</th>
<th>Polyurethane</th>
<th>Perlite</th>
<th>Wood wool</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.000000</td>
<td>0.540517</td>
<td>0.463116</td>
<td>0.516665</td>
<td>0.614607</td>
<td>0.653751</td>
<td>1.00000</td>
<td>0.483414</td>
<td></td>
</tr>
<tr>
<td>RANK EQ.</td>
<td>1</td>
<td>5</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>
For the solution the first-ranked alternative is obtained. First-ranked alternative is styrofoam (Tab. 6.). The diagram that shows a comparison of Vikor solutions for each material is drawn and shown in Fig. 8.

Fig. 4. Comparison of VIKOR results for each materials.

It remains to check sufficiently stable position and advantage over other alternatives (Tab. 7.).

Table 7. Calculating DQ and comparison with \( Q(A^{(2)}) - Q(A^{(1)}) \).

<table>
<thead>
<tr>
<th>QA(2)</th>
<th>QA(1)</th>
<th>QA(2)-QA(1)</th>
<th>DQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4631</td>
<td>0.0000</td>
<td>0.4631</td>
<td>0.1429</td>
</tr>
</tbody>
</table>

Examining the advantage of styrofoam compared to other alternatives, an obtained value DQ=0.1429 means that styrofoam does have a sufficient advantage over other alternatives. The stability of the position is also examined by changing \( v \) value on 0.25 and 0.75.

Table 8. Calculating DQ and comparison with \( Q(A^{(2)}) - Q(A^{(1)}) \).

<table>
<thead>
<tr>
<th>Advantage (check ( Q(A^{(2)}) - Q(A^{(1)}) ) ≥ DQ)</th>
<th>Stability of the position (change ( v ) on 0.25)</th>
<th>Stability of the position (change ( v ) on 0.75)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRUE</td>
<td>TRUE</td>
<td>TRUE</td>
</tr>
</tbody>
</table>

For both changes styrofoam remains the top-ranked so that it can be said that there are enough stable position, shown in Tab. 8. However, with changing the weighting factors, that describe relative preference of each of the criteria, the ranking order of compromise solutions is changing too.

7. Conclusion

Using the specifying problem of warmth buildings, with a practical case study, represented is a scientific method for selecting the alternative choice of insulating materials for building walls using the VIKOR method for multi-criteria optimization.

VIKOR method provides that for selected criteria and their valuation styrofoam should be used for insulation of walls of buildings. With an obtained result, it can be concluded that styrofoam is first-ranked alternative which should be used for warmth buildings.

VIKOR method showed the ease of its application, and as such it can be successfully used in other aspects of decision-making, especially when it is necessary to determine the ranking order of alternatives to enable informed decisions. The approach to ranking priorities based on MCDA provided much better arguments to decision makers enabling them to make informative decisions and not a decision based on some narrow discussions and unjustified
preferences. VIKOR has proven to be very useful in similar prioritizations and has many advantages as compared to other methods, like Promethee, Electre, AHP and others. Nevertheless, other multi-criteria decision aid methods should be tested for their applicability in this problem area.

In addition, besides choosing of insulation of buildings, significant improvement of the distribution of thermal energy process efficiency can be achieved if the warmth of buildings is made by both placing styrofoam and installing new energy efficient windows to the buildings. In that case, the savings of thermal losses and thus saving emission of CO₂ can be calculated, as well as the value of investments and return on investment that would justify such investment. Additional opportunity lays with the installation of an application for predicting outside temperature, in order to eliminate the delay of delivery of thermal energy when there is no need for it.

Research will be continued into the following two directions – first is to evaluate applicability of other multi-criteria decision aid methods in the same problem area and to compare results. Aim would be benchmark appropriateness and quality of the methods, so as time consumed to get the results. Second direction would relate to extending the applied VIKOR method for additional parameters, even the less relevant ones, in order to check for the solution stability. Alternative “Do Nothing” will also be added, in order to check its influence to the achieved ranking.

References


