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Computer Aided Determination of Criteria Priority for Structural Optimization of Technical Systems

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Abstract

In the present paper a choosing of methods for determination of criteria priority (significance) for multicriteria optimization of the structure of technical systems has been done. For this purpose, known methods are analyzed and evaluated against combination of chosen factors of influence. Algorithms and software modules are developed, based on known methods for determination of the weight coefficients' vector, which modules are implemented in a dialog system for structural optimization. Results are shown from the application of the software for solving a particular test problem using the considered methods.

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Keywords: technical systems; multicriteria optimization; structural variant; priority; weight coefficients; algorithms; software

1. Introduction

During the design process of technical systems it is necessary to solve multiple times the problem for choosing optimal (rational, effective) structural variant [1, 2]. The real conditions necessitate this choice to be made according to a combination of criteria, which criteria are very often contradictory. Therefore, the problem for choosing an optimal variant of a technical system represents a multicriteria optimization problem. In a common case the chosen criteria for evaluation of the alternative variants, could have different relative importance (value, priority, significance) depending on the conditions of the particular problem.

Solving of a multicriteria optimization problem relates to a number of difficulties. One of the main difficulties is defining priority for the objective functions [2, 4, 5, 10]. Specialized literature [5-10] analysis shows, that an

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unambiguous choice of suitable methods for determination of priority when solving a certain group of problems cannot be made. Also for a number of known methods for determining the importance of criteria, there is lack of algorithmic and software development, which makes difficult their use in practice. On the other hand, it is known, that as many more solutions of the MOP are produced, while taking into account different importance of the criteria, as much increases the possibility for finding a solution, which satisfies in the best way possible the requirements of the decision maker.

The purpose of the present paper is development of algorithms and software modules of chosen methods for determination of objective functions' priority, which modules will be integrated in a dialog system for structural optimization of technical systems.

Nomenclature A square matrix with binary comparisons α principal right eigenvector n_{max} maximum right eigenvalue number of compared criteria k k^{-th} component of vector α α_k component of row i, j-th column, belonging to a matrix with binary comparisons a_{ij} $A_{\boldsymbol{\Sigma}}$ vector derived from (4) CI consistency index CR consistency ratio RCI random consistency ratio weight coefficient for i-th objective function Wi weight coefficient for i-th objective function Wi В rectangular matrix derived from (9) W weight vector vector derived from (9) i^{-th} objective function i^{-th} objective function vector derived from (12) vector with components $\bar{\alpha}_i$ components of vector $\bar{\alpha}$ defined in (13) R rank of importance IF importance factor PF partial function K set of criteria indexes

2. Choosing of method for determination of criteria priority

The problem for comparing value (significance) of the criteria is one of the main problems when solving multicriteria optimization problems [2, 4, 5, 10]. This problem can be formulated in the following way - find a method for defining priority mathematically and its level of influence on the choice of optimal structural variant of a technical system. This is a complex task, because for its solution a quantitative evaluation of significantly differing objects (the criteria for choosing of optimal structural variant) has to be done, and which criteria are abstract concepts, characterizing the quality of alternative variants.

From the known ways of assigning criteria importance, through priority order, priority vector and weight vector (weight coefficients' vector), the latter is the most commonly used. When assigning a weight vector, for every criterion f_k , $k \in K$, a weight coefficient is assigned (significance coefficient) w_k , $k \in K$. The coefficient w_k is a real positive number. This number w_k defines the relative "weight", "importance", "value" of the k^{-th} criterion in

relation to the other criteria. The weight vector $W = \{w_k\}$, $k \in K$, represents a k dimensional vector, defined in a single hypercube - $W = \{w_k : w_k \in [0,1], \sum_{k \in K} w_k = 1\}$.

In specialized literature are described scores of expert methods for evaluation of objective functions' priority, but a full comparative analysis of all methods is impossible to make. If there was a single universal method, such a comparison could have been made, but such method does not exist. Moreover there are no commonly accepted criteria for evaluation of the methods.

For solving this problem, known literature sources [4-10] were analyzed and the following main factors were classified, having influence over the choosing of a method for evaluation of the criteria priority: complexity of conducting the expert evaluation and labor-consumption for obtaining the expert information; availability of additional information (for instance, information for boundary values of the criteria in the feasible set, etc.); degree of agreement of the experts' opinions; applicability (here under applicability is understood the criteria count, for which the application of a given method is efficient); labor-consumption for processing of the information given by the experts; relationship between the way of defining criteria priority and the used multicriteria optimization methods.

After conducting analysis of over 70 known methods [2] for determination of criteria priority, use of binary comparisons methods is proposed. Nevertheless their labor-consumption, these methods are the simplest and the most justified from a psychological point of view. For the experts it is most convenient in each step to evaluate qualitatively only two criteria. The expert can compare these criteria and to tell which one of them is more important, to give an evaluation of the type "equal importance", "slight superiority of one criterion over other", "strong superiority", etc., but the expert cannot answer how much one criterion is more important than other. In human thinking usually images and words are used, not numbers. Therefore requesting an answer from the expert in the form of a particular number or value representing quantitative equivalent of the level of superiority of one criterion over others, will be very difficult task for the expert.

The binary comparisons methods are characterized with relatively high level of consistency and reliability of the obtained results. For this purpose in some of them are provided corresponding procedures. The so called full or double comparison is applied. The binary comparisons of the criteria are made twice. For example, at the beginning the first and second criteria are compared, the third and fourth criteria are compared and so forth until the last criterion. After that the same procedure is carried out backwards. This provides means for avoiding accidental errors. Moreover, with these methods the final results for the weight coefficients could be obtained by applying the method for consecutive approximation. According to this method, for every following approximation, results from the previous approximation are used as coefficients of significance for the experts' ratiocinations. When certain conditions are met, this process is convergent. The normalized weight coefficients approach some constant values, the latter strictly represent relations between the criteria for given input data.

The main disadvantage of the chosen group of methods is that they are labor-consuming, which is related to the necessity for complex calculation procedures. For overcoming this problem, algorithms and software modules were developed, based on known binary comparisons methods, which are integrated in the software system for multicriteria optimization PolyOptimizer [3].

3. Algorithms of methods for determination of criteria priority

In this paragraph are shown algorithms of known methods for determination of weight coefficients.

3.1. Saaty's method [7]

With Saaty's method a solution of the matrix equation (1) [7] is sought

$$A\alpha = n_{\text{max}}\alpha$$
, where $n_{\text{max}} \ge k$, (1)

where the following algorithm is used:

Step 1. The components of the principal right eigenvector α (column matrix) are defined from (2).

$$\alpha_k = \sqrt[k]{\prod_{j=1}^k a_{kj}} \tag{2}$$

Step 2. The vector α is normalized by the sum of its components (3).

$$\alpha_k = \frac{\alpha_k}{\sum_{i=1}^k \alpha_i} \,. \tag{3}$$

The components of the normalized vector α are the calculated weight coefficients.

Step 3. For determination of n_{max} first the components of vector A_{Σ} (row matrix) are found from (4).

$$A_{\Sigma} = \left\| \sum_{i=1}^{k} a_{i1} \dots \sum_{i=1}^{k} a_{ik} \right\|,\tag{4}$$

Step 4. The vector A_{Σ} is multiplied by the vector α . The result is equal to n_{\max} .

Step 5. The consistency index CI and the consistency ratio CR are calculated from (5) [7].

$$CI = \frac{n_{\text{max}} - k}{k - 1}, \quad CR = \frac{CI}{RCI}$$
 (5)

where *RCI* is given in Table 1.

Step 6. The criterion for accepting the given matrix with binary comparisons is (6) [7].

$$CR \le 0.1$$
.

If CR is greater than 0.1 it is recommended that a reconsideration of the evaluations of the comparisons is to be done, until condition (6) is satisfied.

Table 1. Values of the random consistency index RCI [7].

k	1	2	3	4	5	6	7	8	9	10
RCI	0	0	0.52	0.89	1.11	1.25	1.35	1.4	1.45	1.49

3.2. Pardalos and Mann's method [8]

The method is also known as the "human rationality approach" [8].

Step 1. The matrix of binary comparisons A is composed, having the form (7) [8].

$$A = \begin{vmatrix} 1 & a_{12} & a_{13} & \dots & a_{1k} \\ a_{21} & 1 & a_{23} & \dots & a_{2k} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{k1} & a_{k2} & a_{k3} & \dots & 1 \end{vmatrix},$$
 (7)

where
$$a_{ij} = \frac{w_i}{w_i}$$
, $a_{ij} = \frac{1}{a_{ii}}$, $a_{ij} \in \left\{9,8,7,\ldots,2,1,\frac{1}{2},\ldots,\frac{1}{7},\frac{1}{8},\frac{1}{9}\right\}$, $i, j = 1 \div k$ [8].

Step 2. The matrix equation (8) [8] is composed.

$$BW = b, (8)$$

Where [8]:

$$B = \begin{vmatrix} -1 & a_{12} & 0 & 0 & \dots & 0 \\ -1 & 0 & a_{13} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & -1 & a_{22} & 0 & \dots & 0 \\ 0 & -1 & 0 & a_{23} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & -1 & a_{\frac{k(k-1)}{2},k} \\ 1 & 1 & 1 & 1 & \dots & 1 \end{vmatrix}, W = \begin{vmatrix} w_1 \\ w_2 \\ \vdots \\ w_k \end{vmatrix}, b = \begin{vmatrix} 0 \\ 0 \\ \vdots \\ w_k \end{vmatrix}$$
(9)

Step 3. The linear least squares problem (10) [8] is composed according to (9) [8]:

$$f^{2}(x) = ||b - BW||. ag{10}$$

Step 4. The system of linear equations (11) is composed.

$$\frac{\partial f^{2}(x)}{\partial w_{1}} = 0,$$

$$\frac{\partial f^{2}(x)}{\partial w_{2}} = 0$$

$$\vdots$$

$$\frac{\partial f^{2}(x)}{\partial w_{k}} = 0$$
(11)

Step 5. After solving the linear equations system (11) the components of vector W are found, which components are the sought weight coefficients.

3.3. Voichinskii and Ianson's method [9]

For this method the definition of priority for the criteria is done by one expert, who consecutively makes a qualitative evaluation about the relative importance of all pairs of criteria by using the symbols: "> " - more important, " \approx " - with equal importance, " \prec " - less important. The results are written down in a preference matrix $A = \left[a_{ij} \right]_{k \times k}$. After that the expert must define how many times the importance of the compared criteria differs. Different cases are possible: strongly differing criteria, moderately differing criteria and weakly differing criteria [9].

Determination of the weight coefficients w_i is done in the following order:

Step 1. The symbols \succ , \approx , \prec are replaced in the preference matrix with their corresponding values.

Step 2. The sum $\sum_{j=1}^{k} a_{ij}$ is calculated for every row $i = 1 \div k$ of the preference matrix $A = \left[a_{ij} \right]_{k \times k}$. The results are written as a column vector (12).

$$A^{\Sigma} = \left[\sum_{j=1}^{k} a_{ij}\right]_{k \times 1} \tag{12}$$

Step 3. The values of the coefficients of significance are calculated for all criteria. These values are elements of a column vector $\overline{\alpha} = [\overline{a}_i]_{k \times 1}$, produced by multiplying the preference matrix $A = [a_{ij}]_{k \times k}$ by the column vector

$$A^{\Sigma} = \left[\sum_{j=1}^{k} a_{ij}\right]_{k \ge 1}$$
, i.e. the value of $\overline{\alpha}_i$ is calculated from (13).

$$\overline{\alpha}_i = a_{i1} \sum_{i=1}^k a_{1j} + a_{i2} \sum_{i=1}^k a_{2j} + \dots + a_{ik} \sum_{i=1}^k a_{kj}$$
(13)

Step 4. The values of the normalized weight coefficients of the criteria are calculated using (14).

$$\alpha_i = \frac{\overline{\alpha}_i}{\sum_{i=1}^k \overline{\alpha}_i} \tag{14}$$

3.4. Single comparisons method [6]

- Step 1. The criteria are ordered by decreasing importance (cortege). The most important criterion for the evaluation, staying in the first place of the order, is evaluated as $\frac{f_i^1}{f_i^1} = 1.00$ and with a rank of R = 1.
- Step 2. Every criterion is compared to the preceding by importance and a value for the binary comparison between the two criteria is defined (15).

$$\frac{f_i^R}{f_j^{R-1}} \in [0.1, 0.2, \dots, 1.0], \ i, j, R \in [1; k]$$
(15)

Step 3. The significance coefficient for every criterion is calculated (16).

$$IF = \prod_{R=[2:k]}^{2} \frac{f_i^R}{f_i^{R-1}} \tag{16}$$

Step 4. The significance coefficients are normalized and so the weight coefficients are obtained.

4. Software development

On the basis of the developed algorithms and software modules for determination of criteria priority for multicriteria optimization problems related to technical systems, a graphical user interface has been developed, which is integrated in the software system PolyOptimizer [3].

The user interface provides the decision maker, with an easy and clear way to define and input his/her subjective preference towards a given target function (criterion). The aim during development of the graphical user interface for the priority definition modules, has been for maximum abstraction of the user from purely quantifying the priority, i.e. concrete value, instead to the user is presented a visual way, through which he/she can express his/her preference. This is achieved through interactive scales and symbols. Eliminating the deterministic nature of the priority definition, aids the decision maker when making a choice, because in most cases the user has difficulties when asked to give a particular value for the priority of one criterion over other (or over the rest criteria).

Saaty's method, also known as method for hierarchy analysis, can be used for defining criteria priority for problems with three to ten objective functions. For input of the binary comparison between two objective functions horizontal scales are used. On the two sides of each scale there are corresponding objective functions. The position of the scale's indicator can be changed. The meaning of the indicator's position is as follows: as close to a given objective function the indicator is positioned, as much that objective function is preferred over the other objective function corresponding to the given scale.

Saaty's method checks for consistency of the binary comparisons' matrix. When consistency is lacking, i.e. there is contradiction in the input information, Saaty's method proposes change of certain priorities, so that the problem becomes consistent. After the change, the newly entered information must be recalculated.

The user interface for configuring input data for Saaty's method is shown on Fig. 1.

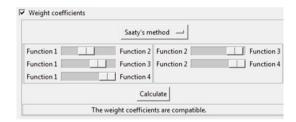


Fig. 1. User interface for Saaty's method.

The Pardalos and Mann's method can be used for the same number of functions, as Saaty's method. The user interface is identical with that used for Saaty's method and the working principle is the same. First the Saaty method is used for removing any inconsistencies in the input information, and after that, the weight coefficients are calculated. The Pardalos and Mann's method is characterized by the interpretation, that when minimizing regret, losses or raising income, the decision maker is aiming for error minimization when evaluating binary comparisons.

Voichinskii and Ianson's method offers to the user a possibility for evaluation by two indications: relative importance and difference in the significance of the criteria. The method is characterized by its simplicity and the possibility for taking in account the degree of importance between the compared criteria. On Fig. 2a is shown the user interface when using this method.

For input of the relative importance the user uses the button " \approx ". When this button is pressed a menu appears, from which the relative importance between the two criteria can be chosen. The symbolic assignments are as follows: "<" - the objective function from the left side of the symbol is less important from the function that is on the right side of the symbol; " \approx " - the two objective functions are equally important; ">" - the objective function from the left side of the symbol is more important from the function that is on the right side of the symbol.

The difference in the significance of the compared target functions is assigned through the vertical scales. The down position of the indicator means "slightly differing criteria"; positioning of the indicator in the middle of the scale means "moderately differing criteria"; the up position of the indicator means "significantly differing criteria".

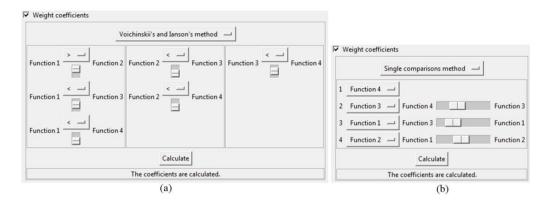


Fig. 2. User interface for Voichinskii and Ianson's method (a) and for the single comparisons method (b).

For the single comparisons method the decision maker arranges the objective functions in a cortege, in which the most important function is first, followed by the second most important and so forth. Then each target function is compared with the previous one in the cortege and binary comparisons are formed. The user interface when using the single comparisons method is shown on Fig. 2b.

5. Example

For given data concerning criteria importance and the ordering $f_4 > f_3 > f_1 > f_2$, determine the weight coefficients' values by using the examined methods, and find the corresponding structural variants of the TS, which are Pareto optimal for the following multicriteria problem:

$$\min f_1(x) = \sum_{n=1}^8 f_1(x_n^l), \min f_2(x) = \sum_{n=1}^8 f_2(x_n^l), \min f_3(x) = \sum_{n=1}^8 f_3(x_n^l), \min f_4(x) = \sum_{n=1}^8 f_4(x_n^l)$$
(17)

satisfying the constraints (18).

$$g_1(x) \le 9.05, g_2(x) \ge 12, g_3(x) \le 64$$
 (18)

Values for the objective functions and the constraints are given in tabular form (Table 2). With F_n , $n=1\div 8$ are marked the partial functions of the technical system (table rows), which the system must realize. Each partial function is executed from a definite set of alternative devices $x_n^l \in X_n = \{x_n^1, x_n^2, ..., x_n^{l_n}\}$ [3]. For example the first partial function F_1 is executed from three alternative devices, the second F_2 and third F_3 one are executed also from three devices, the fourth F_4 from four devices and etc. In each cell of the table are given seven values, representing the values and parameters of the corresponding elementary device x_n^l , for which parameters optimal values are sought $f_k(x_n^l)$, $k=1\div 4$ or there are imposed constraints $g_m(x_n^l)$, $m=1\div 3$. There are no constraints concerning compatibility between the elementary devices. Therefore the number of possible variants for building the system is $3\times 3\times 3\times 4\times 4\times 7\times 10\times 9=272160$.

Table 2. Values of the objective functions and the constraints.

PF	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X ₉	X_{10}
\mathbf{F}_{1}	78;129;89	19;42;46	130;66;103							
	64;1.179	48;1.164	39;1.051							
	0.395;10	1.429;7	0.203;6							
F_2	102;118;92	51;45;35	76;20;40							
	38;1.055	79;1.052	48;1.073							
	0.582;5	1.140;10	1.790;7							
F_3	96;106;96	36;20;66	27;98;62							
	69;1.174	130;1.044	78;1.110							
	2.134;8	1.553;7	1.607;9							
F_4	33;42;109	111;106;27	86;107;69	71;89;71						
	56;1.087	103;1.040	52;1.105	43;1.021						
	2.817;9	2.329;7	2.089;9	2.634;9						
F_5	84;77;117	32;77;49	117;81;46	41;108;88						
	48;1.179	101;1.071	27;1.122	90;1.107						
	0.651;7	1.941;9	2.594;9	0.298;7						
F_6	31;129;88	49;104;35	25;45;78	29;36;80	46;14;123	36;41;116	45;116;32			
	14;1.189	55;1.019	102;1.162	19;1.128	116;1.148	45;1.165	71;1.147			
	2.083;5	0.857;9	2.675;6	2.572;9	1.955;6	2.387;6	0.512;10			
\mathbf{F}_{7}	102;63;84	23;13;127	15;22;34	25;53;110	48;60;34	115;128;85	18;86;33	43;60;54	49;10;71	93;106;17
	56;1.189	50;1.217	62;1.075	114;1.207	68;1.131	39;1.082	81;1.155	98;1.203	76;1.129	11;1.17
	1.754;10	2.045;8	2.878;6	1.287;10	1.671;7	0.789;8	0.285;9	2.145;6	1.11;10	2.02;7
F_8	97;110;72	10;95;76	46;104;126	93;73;77	78;16;62	49;84;21	126;43;77	47;84;11	114;20;92	
	46;1.005	99;1.018	87;1.129	121;1.165	79;1.018	87;1.074	31;1.127	86;1.056	73;1.117	
	1.370;7	1.156;10	1.833;9	0.487;6	0.347;5	0.360;10	0.631;10	2.970;8	1.256;9	

The calculated weight coefficients α_k , $k = 1 \div 4$ for each criterion are shown in Table 3.

Table 3. Priority vectors calculated by using the developed software modules.

Method	α_1	α_2	α_3	α_4
Saaty	0.07	0.04	0.26	0.63
Pardalos and Mann	0.06	0.04	0.19	0.72
Voichinskii and Ianson	0.14	0.10	0.32	0.44
Single comparisons	0.12	0.02	0.24	0.63

The obtained results show that the coefficients' values calculated by the methods used are differing, although insignificantly, while the cortege (priority ordering) is preserved for the objective functions. These differences are explained by the different solving approaches used in each method. This gives an additional possibility for thorough and precise study of the possible solutions of multicriteria optimization problems, because small changes in the weight coefficients' values lead to choosing of different Pareto optimal structural variants of the TS [8]. As a confirmation of this statement, problem (17) and (18) is solved with the obtained weight coefficients. The results are shown on Fig. 3, where w1, w2, w3 and w4 are relative deviations from the optimum of each criterion when solving problem (17) and (18). For comparison the last solution (Compromise) is obtained for equal importance (without

priority) of all compared criteria. Determination of the weight coefficients is of significance for finding an optimal structural variant of TS.

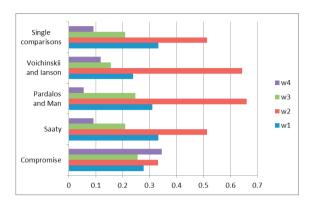


Fig. 3. Solutions of the problem with different values for the weight coefficients.

6. Conclusion

Factors influencing the choice of method for determination of criteria priority, when solving multicriteria optimization problems, have been systematized. Taking into account these factors, a group of methods has been chosen which group is of methods for binary comparisons. They are realized under the form of software modules, implemented in the dialog system for multicriteria optimization PolyOptimizer. The developed software modules provide the possibility for a deep and precise examination of the possible solutions of multicriteria optimization problems, by varying the weight coefficients' values. In that way, the chances for finding a solution, which satisfies in the best possible way the requirements of the decision maker, are increased. The results from this development will aid the designers of complex technical systems in their work, when they are choosing an optimal variant by decreasing the time needed for decision making.

The future development of the dialog system will be in terms of implementing additional modules for determination of weight coefficients through application of ranking methods, building of the Pareto front without the need of calculating all possible combinations (brute force approach) and solving of problems for choosing of optimal structural variant under the conditions of incomplete information.

References

- [1] Boyadjiev, I., I. Malakov. Choice of an optimal structure option of a technical system in conditions of incomplete information. Annals of DAAAM for 2002 & Proceedings of the 13th International DAAAM Symposium, ISBN 3-901509-29-1, Vienna, Austria, 23÷26.10.2002, Published by DAAAM International, Vienna, Austria 2002, pp 059÷060.
- [2] Malakov, I. Methodology for choice of optimal assembly structural variant. (in Bulgarian) Research work qualifying to receive the title of Professor, Sofia, 2009.
- [3] Malakov, I. & Zaharinov, V. Interactive Software System for Multicriteria Choosing of the Structural Variant of Complex Technical Systems, Annals of DAAAM for 2012 & Proceedings of the 23rd International DAAAM Symposium, ISBN 978-3-901509-91-9, ISSN 2304-1382, pp 0199 - 0204, Katalinic, B. (Ed.), Published by DAAAM International, Vienna, Austria 2012.
- [4] Mihalevich, V. & Volkovich V. (1982) Computational methods for study and design of complex systems. (in Russian) Nauka, Moscow.
- [5] Riabacke, M., Danielson M. & Ekenberg L. State-of-the-Art Prescriptive Criteria Weight Elicitation. Advances in Decision Sciences, vol. 2012, Article ID 276584, 24 pages, 2012, doi:10.1155/2012/276584.
- [6] Rinza, P., H. Schmitz. Nutzwert-Kosten Analyse. VDI Verlag GmbH, Duesseldorf, 1977.
- [7] Saaty, T.L. Eigenvector, and logarithmic least squares. Eur. J. Oper. Res. 1990. V. 48, №1, pp. 156÷160.
- [8] Triantaphyllou, E. Multi-criteria decision making methods. A comparative study. Kluwer Academic Publishers, Dodrecht, 2000.
- [9] Voichinskii, A.M., E. J. Ianson. Technology of products for measuring equipment. (in Russian) Leningrad, Mashinostroenie, 1988.
- [10] Zaichenko, U.P. Operations study. (in Russian) Kiev, Vuisha shkola, 1988.