Suitability of Pulse Error Models for Time-Domain Spectrum Measurements

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Abstract

Measurements of electromagnetic compatibility properties of electronic systems are inevitable to guarantee the reliability and safety of control or robotic systems. For automation of such measurements time effective TDEMI (Time Domain ElectroMagnetic Interference) devices are used. Their principally new structure with parallel analog input channels followed by fast digital signal processing provides fast EMI spectrum measurements. On the other side it brings the risk of introduction of new unknown error sources caused by differences between transfer characteristics of parallel input channels. A special pulse model of the errors was proposed. Results of the precise simulation pulse model and simplified theoretical pulse model are compared together and with the experimental test on the harmonic input signal.

Keywords: Multiresolution quantization; noise floor; spectrum measurement; TDEMI

1. Introduction

Electromagnetic interference (EMI) is significant problem of the electrical and electronic equipment design. Especially in areas where a fault of the component would cause a huge damage, economical losses or even endanger human health a special care has to be paid to the electromagnetic compatibility testing. EMI is actual for mechatronic systems represented e.g. by modern cars, for the automation and control equipment where a fault of one
component influences functionality of huge facilities or production. The issue is important also for sensitive precise robotic systems.

EMI measurements are traditionally performed by conventional analog EMI receivers or spectrum analyzers operating in the frequency domain. Such devices are based on a superheterodyne principle. The receiver has to sweep over the desired frequency range. Therefore it provides information only about one spectral component at a time. The advantage of this principle is a high dynamic range of the measurement required by EMI standards. However such measurement is slow as it is not possible to use fast sweeping times because the settling of the output of a detector situated at the end of the measuring channel should be enabled.

From the economic point of view it is reasonable to investigate methods that lead to the reduction of the spectrum measurement time. Automation helps to optimize a measurement process [1]; however it is limited by the performance of the measuring settlement. New and fast testing strategies should support the idea of just in time production [2]. Time domain EMI (TDEMI) systems offer distinct advantages in comparison with conventional receivers [3]. They provide information about all spectral components of the EMI signal at the same time using short time Fast Fourier transform. The technology of multiresolution quantization analog-to-digital converters (MQADC) used inside a TDEMI unit enables the high dynamic range of fast spectrum measurements.

The MQADC technology employs several parallel input channels and switching between their outputs to avoid the degradation of the dynamic range caused by the low bit resolution of fast analog-to-digital converters (ADC). Some influence of the quantization process on measurement results remains also for the MQADC. However practical experience shows that there are more serious sources of errors in real systems. It is hardly possible to avoid the offset or phase shift between parallel input channels likewise the gain error differences. So serious signal discontinuities could arise in points where the system switches from the output of one channel to another [4]. Spurious spectrum components generated by those discontinuities significantly restrict the spurious free dynamic range of a real TDEMI. The problem is similar to ADC systems with time interleaving or with reconfiguration structure [5] where offset or gain mismatch of parallel channels degrades overall performance.

The model of the error of a two channel MQADC system is discussed in the paper suitable for the analysis of the impact of signal discontinuities on the measured spectrum. A harmonic input signal may be considered as suitable for modeling the measured interference of devices operating on the switched mode power supply principle, where the disturbance is like a mixture of sinusoids. For the harmonic input signal differences between channels result in disturbances similar to time-domain error pulses. The erroneous waveform could be simply modeled as an additive impulsive signal. Results presented in this paper were obtained from simulations based on this pulse model of error. For the analytical expression of the error a simplified model based on rectangular pulses was designed. Fundamental parameters of the error were taken from experiments and results have been confronted with experimental spectra.

2. Technology behind TDEMI

Nowadays TDEMI systems may be used for conducted as well as radiated emission measurements. These devices require a high speed floating point ADC to reach sufficiently fast sampling frequency of the input EMI signal and a very powerful digital signal processing module being able to perform Fast Fourier transform calculations during microseconds [6]. A floating point ADC is necessary for sufficient dynamic range of measurement. As a floating point ADC the multiresolution quantization system is employed within the TDEMI. It consists of several high-speed flash ADCs providing the sufficient sampling rate in the range of gigasamples per second.

The measurement channel of MQADC consists of power splitter, several parallel channels with different gains, separate ADCs and one common block of digital signal processing. The principal block structure of the MQADC was depicted e.g. in [4]. The power splitter distributes the analog signal to all parallel paths. Recently 3 channels are usual. Separate amplifiers/attenuators provide the different range and voltage resolution of individual channels. All channels are simultaneously sampled and converted by the identical 8 or 10 bit very fast flash ADCs. The final discrete value is created by extracting the output from that ADC offering the best resolution but with the range still covering the actual input value. In a TDEMI unit short time Fast Fourier transform is finally applied to the sampled data [7].
Even if perfectly realized the quantization system like the MQADC generates error components in the spectrum of e.g. harmonic signal [8][9] due to the quantization error. However, in real systems there are more serious sources of errors. For a successful reconstruction of the measured sampled values in a TDEMI system, it is necessary to know properties of analog input circuits all over the frequency range. These properties have to be reflected in the digital processing module. Channel mismatched compensation techniques were developed especially for time interleaved ADCs [10]. On the other hand, publications describing properties of TDEMI measurement systems are dealing mostly with perfectly matched power splitter and digital processing module. The aim of this paper is to analyze impact of mismatch error on spectrum measured by the TDEMI system.

The main task of the analog input circuits is to split one analog signal into multiple paths with precisely set gains. This is a nontrivial task because ideally the uniform amplification level should be achieved for the frequency range up to several GHz without introducing a different phase shift between channels. In a real system amplitude and phase frequency characteristics in each channel are not perfectly matched in the whole frequency range. Moreover it is hardly possible to avoid a slope difference between channels. Inside a TDEMI device the time domain signal is composed from samples received from different parallel channels. For each time or a time interval only samples from one channel are used.

So serious signal discontinuities arise in points where the system switches from one ADC output to another. For harmonic input signal (frequency $f=175$ kHz and the amplitude $A=0.128$ V) an example of a time representation of the signal used for further processing inside a real TDEMI was presented in [4]. The digital signal processing part calculates Discrete Fourier transform from samples to obtain the signal spectrum. Then, significant spurious components arise in the measured spectrum as could be seen in Fig. 1 obtained for the mentioned harmonic input signal. Spurious components generated by waveform discontinuities significantly restrict the spurious free dynamic range of the real TDEMI device for continues input signal types.

![Fig. 1. Error components of the experimental spectrum measured by a TDEMI unit for the harmonic input signal of the frequency $f=175$ kHz.](image)

3. Pulse error model

Experiments suggest that main spurious components of TDEMI measurement results are caused by discontinuities of the time domain signal representation. Such discontinuities present in the waveform reconstructed from the sequence of samples obtained by the MQADC could be modeled as an additive impulsive error signal. The principle is obvious from Fig. 2. As the spectrum of the sum of two signals is just the sum of both signal spectra the spectrum of the impulse disturbance could be used to precisely model the spurious spectral components. This is the fundamental assumption used in our analysis. The model will be used for simulation and also for theoretical analysis of errors to demonstrate that the discussed error type is the dominant source of spurious components visible in Fig. 1.

Simulation model consistently follows the idea of pulse error model mention above and outlined in Fig. 2. Two error pulses of the model per period of the input signal enable precise estimation of the real error if proper shapes of
pulse-forming waveforms are used. For a harmonic input signal the harmonic waveform with the right offset, gain and phase composes pulses for such accurate simulation model. Compared to a real system the model neglects quantization error, however, it is precise for the estimation of the distortion caused by the discrepancy between channels and could be used as a reference for other approximate models.

On the other hand only simple rectangular pulses will be used to reduce complexity of mathematical expressions for further analytical investigation based on a model theory. The voltage level of every pulse will be calculated by averaging of the original values within the error pulse in the mathematical AVE (AVEraging) model explained below.

![Fig. 2. Basic idea of the pulse error model.](image)

### 3.1. AVE model

An analytical expression of the pulse model of spectral error components could be useful for understanding of the error behavior and for further theoretical considerations. We will assume that the input test signal is harmonic. Simplifications in a derivation of a mathematical model allow obtaining convenient analytical expressions. Therefore, in this case, only rectangular pulses with amplitudes $U_1$ and $U_2$ will be used for the error waveform description, although they do not describe the real error pulse shape perfectly (see Fig. 2). Voltage levels $U_1$, $U_2$ will be calculated by averaging of real error values within every active interval of a pulse. Such AVE model should be still precise if the offset error is dominant in the MQADC.

As a spectrum of an ideal harmonic signal contains only one frequency component the higher harmonics of the error pulse signal could be regarded as theoretical spurious components present in the measured spectrum. For the evaluation of the estimation of error spectral components we need at first to find the analytical expression for harmonic components of a periodic square pulse. Let us consider a periodic square pulse signal with the amplitude $U$, the time of duration $T_p$, the center time $t_c$ and the period of repetition $T$. Fourier coefficients for this pulse signal are (expressions for $n$-th coefficient, $n$ is integer)

$$A_{p0}(T, U, T_p) = U \frac{T_p}{T}$$  \hspace{0.5cm} (1)

$$A_{pn}(T, U, T_p, t_c) = \frac{U}{n\pi} \cos\left(\frac{n\pi}{T_p} \frac{2t_c}{T}\right) \sin\left(\frac{n\pi}{T_p} \frac{T_p}{T}\right) - j \frac{U}{n\pi} \sin\left(\frac{n\pi}{T} \frac{T_p}{T}\right) \sin\left(\frac{n\pi}{T} \frac{2t_c}{T}\right) \sin\left(\frac{n\pi}{T} \frac{T_p}{T}\right); n > 0$$  \hspace{1cm} (2)

As outlined in Fig. 2 (but considering only rectangular pulses) the error waveform is a sum of two pulse signals, which have different time shifts of pulses and different amplitudes. Utilizing the linearity of Fourier transform the final spectrum of the signal composed from multiple parts can be calculated as a sum of partial spectra calculated from each partial signal separately. Therefore spectral components of the error waveform are sums (index 1 and 2 means the first and the second pulse respectively)
Eqs. (3)(4) after substitution from (1)(2) already express spectrum of error estimated by mathematical AVE model. However, simplified notation of the model is possible. We will consider that the block of signal processing switches between channels according to data obtained from the channel 1. For our analyses we will regard the signal from this first channel as to be ideal, i.e. this is the reference waveform. For such case the center of the first error pulse is placed at the beginning of the given period $t_{c,1} = 0$ while the second pulse lies in the middle $t_{c,2} = T/2$. The width of pulses is the same $T_p = T_{p,1} = T_{p,2}$. From (2) one can see that imaginary part of the spectrum will be zero and finally we can derive for the error model

\[
A_{e,0} = A_{p,0}(T, U_1, T_{p,1}) + A_{p,0}(T, U_2, T_{p,2})
\]

(3)

\[
A_{e,n} = A_{p,n}(T, U_1, T_{p,1}, t_{c,1}) + A_{p,n}(T, U_2, T_{p,2}, t_{c,2}) ; n > 0
\]

(4)

In the AVE model we assume that the time domain signal is composed from three components – the non-distorted harmonic signal at the input of the system and two square wave signals that represent the error signal. For such simplified representation of the error the composed signal does not fit perfectly the waveform observed at the MQADC output if there is a gain and phase error. However the model offers a simple method for analyzing the spectral coefficients analytically while it is should be precise for offset error as the error waveform caused by only offset between channels is actually of rectangular shape.

4. Simulations and discussions

A simulation pulse model and the simplified mathematical AVE model were proposed that match the behavior of most significant errors that happen in the analog part of a TDEMI system. We would like to present that those models are suitable for the further examination of errors introduced by analog circuits of the MQADC system. The simulation model should precisely estimate spurious components present in the measured spectrum caused by discrepancies of input channels. The AVE model enables further analysis but its accuracy is limited and influenced by the error properties. We have tested properties of both models using error parameters of the experiment mentioned in chapter 2.

According to the experimental waveform from [4] we have identified parameters of the error after applying some simplifications. The signal from channel 1 is regarded as to be ideal, so expression (5)(6) could be used as theoretical model. For the harmonic input signal of the frequency $f=175$ kHz and amplitude $128$ mV the offset between channels is $6$ mV, gain of the second channel is $G = 0.83$ and phase between channels is $\phi = -4^\circ$. Constant threshold level between channels $60$ mV is used in models representing the voltage where the system switches from the first to the second channel or vice versa. For a known input signal this threshold determines duration of each error pulse which is $T_p=1.970$ $\mu$s for this case. Voltage levels $U_1$ and $U_2$ entering into the AVE model were calculated as mean value of simulated voltage waveforms inside the pulses.

In the first stage we draw time waveforms created according the principles of motels. In Fig. 3a MQADC output signal is depicted (sum of original signal and error) obtained from proposed models. The simulated signal is drawn with the black dash-dot line for previously identified parameters. However the simulation or AVE model enables to modify error parameters. So, the other waveforms displayed in Fig. 3a represent the MQADC output waveform for no gain discrepancy and phases $\phi = -4^\circ$ and $\phi = 0^\circ$ respectively. For the case of offset error only ($G=1$, $\phi = 0$) there is no difference between simulation and AVE model (gray waveform).
Fig. 3. (a) modeled waveforms – the items of legend mean: phase; gain; S-simulation/AVE model; (b) error pulses for ideal gains $G=1$ and changing phase between channels $\phi = 0^\circ; -1^\circ; -2^\circ; -4^\circ$: simulation – solid lines; AVE model – dotted line.

As we have suggested the models allow investigation of several combinations of error parameters. We are trying to test impact of modifications in gain and phase error keeping offset error and threshold voltage constant. In Fig. 3b error waveforms are shown obtained for ideal gain $G=1$ and various phases $\phi = 0^\circ; -1^\circ; -2^\circ; -4^\circ$. Solid lines correspond to simulation model. The dotted line represents AVE model for $G=1$ and $\phi = -4^\circ$. AVE model reacts inadequate to changing phase. Therefore the theoretical error waveform for $\phi = -4^\circ$ stays almost the same as for $\phi = 0^\circ$. This has reflected also in TDEMI output waveforms from Fig. 3a. In detailed window one can notice, that the waveform composed from original signal and AVE model for $\phi = -4^\circ$ is closer to simulated waveform for $\phi = 0^\circ$ than for $\phi = -4^\circ$.

Fig. 4. Spectra for ideal gain $G=1$ and different phases $\phi = 0^\circ; -1^\circ; -2^\circ; -4^\circ$: (a) simulation; (b) AVE model.

The purpose of both models is to estimate error spectral components. In previous graphs we have only demonstrated the problem of simplified AVE model in the time domain. Based on previous considerations we expect better fidelity of the simulation model. In the following graphs we will compare error spectra resulting from both models. Spectrums corresponding to described error properties $G=1$ and $\phi = 0^\circ; -1^\circ; -2^\circ; -4^\circ$ are depicted in Fig. 4a for simulation and in Fig. 4b for theoretical model. One can see that in simulated results the third and higher odd harmonics rise rapidly already for little $\phi$. On the other hand the AVE model estimates precisely only even harmonics caused by offset between channels. Even harmonics actually does not change with the phase. The odd components of the mathematical AVE model related to the phase between channels achieve too small values.
To study also the influence of the gain error upon the error spectrum we used ideal phase $\varphi = 0$ and changing gain $G = 1; 0.95; 0.9; 0.83$ for the next set of spectra. In Fig. 5a simulated results are depicted while theoretical results are shown in Fig. 5b. Changes of gain influence not only simulation but also AVE model results. Reaction of the AVE to changing gain looks better than previous response to changing phase. However, this mathematical model still underestimates the third harmonic while it now overestimates other odd harmonics.

Finally we have investigated again the case of changing phase but now for both offset and gain values close to experimental conditions. The gain error is dominant therefore there is only little change caused by phase shift in simulation results depicted in Fig. 6a. Important outcome here is the similarity of simulated spectra with experimental from Fig.1. This confirms correctness of chosen path to model the dominant spurious components using idea of error time-domain pulses. Of course the accuracy of the simplified AVE model is worse again. For such big gain discrepancy $G = 0.83$ the mathematical model seems not to be affected by considered phase changes $\varphi = 0^\circ; -1^\circ; -2^\circ; -4^\circ$. This is obvious from Fig. 6b where the theoretical spectra are compared with the simulation result (gray line, $\varphi = 0^\circ$).

**Fig. 5. Spectra for zero phase and different gains $G = 1.00; 0.95; 0.90; 0.83$: (a) simulation; (b) AVE model.**

**Fig. 6. Spectra for the gain $G = 0.83$ and different phases $\varphi = 0^\circ; -1^\circ; -2^\circ; -4^\circ$: (a) simulation; (b) AVE model compared with simulation for $\varphi = 0^\circ$ (gray line).**

**5. Conclusion**

Fast spectrum measurement equipments like TDEMI systems seem to be an important alternative technology to conventional EMI measurement systems. The spectacular hardware structure of the measuring channel and
advantages of digital signal processing offer improved properties and accelerate testing of parts and circuits used inside control, robotic or other systems during the production. However, spurious components arise in the measured spectrum for signals crossing range of subsequent channels. In order to look for solutions and methods for suppression of the disturbance it is necessary at first to identify the dominant reason of the problem and to find appropriate tool for further analysis.

A special pulse model of error was designed which estimates spectral error components of a TDEMI device caused by discrepancy between input channels. The simulation pulse model was tested in comparison with the experimental spectrum and with the theoretical model derived for the harmonic input test signal employing two channels. It was demonstrated that discrepancies of input channels represent the dominant error in the experimental spectrum.

The approximate mathematical model is based on simple rectangular pulses using averaging of voltage values inside the pulse – AVE model. It is accurate for prevailing offset between channels. It does not respond adequately to the phase shift between channels. Also in the case of the gain error the mathematical model could be inaccurate. As the third harmonic becomes the dominant spurious component with a growing phase or gain error an improved analytical model should be proposed which will adequately involve both parameters.

For the next investigation we have find a reference model of error which is the simulation pulse model. For rough estimate of spurious component or in the cases of dominant offset error also the proposed AVE model could be eligible. However, for rigorous theoretical analysis improved mathematical model should be found. Further experiments in wider frequency range of the input harmonic test signal will help to identify possible offset, gain and phase error combinations and intervals. Also different signal shapes may be considered. Those steps of identification of the error behavior will indicate the direction for development of error suppression techniques.

Acknowledgements

Work presented in this paper was supported by the Slovak Research and Development Agency under grant No. APVV-0333-11 and by the Slovak Grant Agency VEGA under grant No. 1/0963/12.

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