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## Application of the Optimal Control Problem in New Product Launching Process

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### Abstract

The optimal control modeling of the economic problems of new product launching has the advantage of giving interpretation the best results and their use in finding the most appropriate choices of four constants that appear in the equations of state. The equations of state have been used and proposed by other researchers in the field, but they have avoided the optimal control theory, considering it too laborious and addressing the issue of the release of a new product on the other mathematical ways. The aim of the study is to propose an optimal control problem specifically to the product launch process which can be used in product promotion process. We suppose that the company has three objectives: the first is to maximize the product image (the goodwill) at launching time T for minimize the advertising cost. The second objective is to actuate the launch – the launching time T, and the third objective is to plan the launch and the advertising campaign on the  $[0, T]$  interval.

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### 1. Introduction

Mathematical modeling of the new products launching process can be made through several mathematical theories. Such models are: optimal control theory, nonlinear mathematical programming and stochastic theory. [1]

We will limit the given model approach "optimal control" that the theoretically is more laborious but provide efficient numerical methods and easier to interpret in terms of applicants.

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## 2. Economical model description

The life cycle of a product from its development and the perceived need to eliminate from the market, comprises a succession of phases taking place in three main stages: creation, production / distribution, product removal.

During the product creation phase exists only in imagination. After growing market study product design stage, through different phases: feasibility study, concept, design, prototyping, industrialization.

*Launching.* Product release phase begins when the product is placed on the market when sales are low and negligible profits. The main objective of the company is the consumer information about the appearance of new product for it to gain a place in the market. The company ensures that both buyer and vendor to know the product and its benefits.[2], [3]

Due to the difficulties faced by companies during the launch - only some of them have the resources, technological capacity and market information needed to successfully launch new product - many new products can fail, failing to go beyond this stage.

*Growth.* Once successfully launched, the new product will begin to record rapid growth in sales which, together with the relative lack of competition, this stage can be the most profitable in the entire product life cycle. In the stage of growth, marketing expenses begin to decline, although they continue to be maintained at a relatively high level.

Growth stage's goal is to maximize product market share and create a strong brand. Achieving this goal requires:

- To maximize the market demand;
- Quantifying consumer demand depending on orientation similar products launched by competitors;
- Range of products, expanding distribution and boost brand preference;
- Identification of new market segments that product, once modified and differentiated to be sold.

As a result of competitive pressures that begin to appear in the stage of growth, the company will begin to cut prices higher launch.

*Maturity.* Maturity stage sales volume stabilizes at consumer customers, who find the appropriate product needs, preferences and their motivations. The company seeks to maintain competitive advantage by improving product characteristics, extending warranty and after-sales services, discounts etc.

Competition reaches the highest level by focusing on the market share and involve higher costs and promotional discounts for intermediaries etc. Actions taken by the company are designed to ensure efficient distribution to maintain customer loyalty for the brand and product. Extending the maturity stage of the product can be achieved by identifying new ways to use the product, thus maintaining its utility and allowing access to new market segments.

*Decline.* Physical wear and / or moral, changing consumer attitudes toward the product, familiarity with the product, the emergence of substitute products or the emergence of a more powerful product on the market are causes that can cause a decrease in profitability, which is reflected in the decline of the product - perhaps irreversibly.

In order to eliminate the drawbacks of each stage of the product life cycle, the company must realize a balanced product portfolio, consisting of products at different stages of the life cycle to compensate for any losses caused by "aging" simultaneous multiple components products mix. Therefore, the company should have a range of products with a complex product so as to include "young" and "mature", dealing products "aging" in a manner that will not affect the company's future projects.

## 3. The formulation of control optimal problem for new product launching

In this mathematical model will use the following basic concepts:[4]

► A function  $x(t)$ ,  $t \in [0, T]$ , called „goodwill function”, impression of new products launched on the market. The value  $x(t) \in \mathfrak{R}$ , is measured in percentage (%).

► A function  $y(t)$ ,  $t \in [0, T]$ , called “the cost function until  $t \in [0, T]$ ”, costs made by the beneficiary to launch

new products on the market. The value  $y(t)$  is measured in EURO.

► A function  $u(t)$ ,  $t \in [0, T]$ , called “the control function”, the value  $u(t)$  is measured in EURO. This function (pump) adjusts costs at  $t \in [0, T]$ .

► The time  $[0, T]$  is the maximum amount proposed by the beneficiary for the new product launch.

State equations governing this process is a generalization of the equations formulated for the first time by Nerlove-Arrow. [5][6]

The state equation system is:

$$\begin{cases} x'(t) = b \cdot u^\alpha(t) - c \cdot x(t) \\ y'(t) = u(t) \cdot e^{-pt} \end{cases}, t \in (0, T) \quad (1)$$

Constants appearing in (1) are factors regulating dynamic system that has the following interpretation:

$b$  = coefficient which represents the influence of increasing or decreasing the control function  $u(t)$  determined by increasing or decreasing function of “goodwill”  $x(t)$

$c$  = coefficient which determines the “goodwill”  $x(t)$  decreasing

$p$  = exponential coefficient of variation that balances the costs of promoting  $y(t)$  expressed through control function  $u(t)$  so that there is a correlation between the function  $x(t)$  a “good impression” and the function  $y(t)$  promotion expenses time  $t$ .

$\alpha \in (0, 1)$  is a coefficient that defines the influence of external market.

About the control  $u(t)$  will be assumed in all cases that is bounded.

In our case we obtain two optimal control problems, namely:

- Optimal control problem with fixed ends
- Optimal control problem with mobile extremities.

Control problem with fixed ends means that the system (1) we associate the conditions:

$$\begin{cases} x(0) = x_0, y(0) = 0 \\ u(t) \in [0, u_0] \end{cases}, \forall t \in [0, T] \quad (2)$$

and are asked to determine the minimum  $t^*$  so as to  $t^* \in (0, T]$  and

$$\begin{cases} x(t^*) = \bar{x} \\ y(t^*) = \bar{y} \end{cases} \quad (3)$$

Consequently, all systems (1) + (2) + (3) form optimal control problem with fixed ends. If  $(x^*(t), y^*(t))$ ,  $t \in [0, t^*]$  is the solution of this problem, in terms of geometry is a curve joining the fixed points  $A(x_0, 0)$  and  $B(\bar{x}, \bar{y})$ . To note is that the numbers  $x_0, \bar{x}$  and  $\bar{y}$  are given as the constants  $\alpha, p, b, c$ , respectively  $u_0$ .

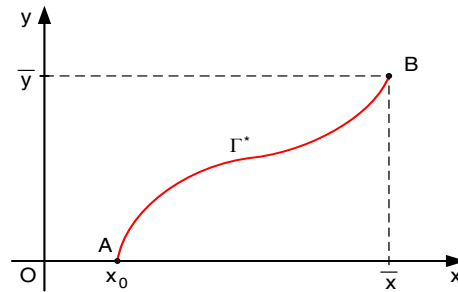


Fig.1. The optimal trajectory  $\Gamma^*$ .

In Figure 1 we note with  $\Gamma^*$  the curve image of the optimal control problem with fixed ends which means that the costs are 0 and the goodwill is known at the beginning of new product launching process.

#### 4. Conclusion

The formulation of the product launching problem is realised correctly and the main contributions of the authors are:

- Design an original mathematical model for synthesis and optimization of launching a new product using optimal control theory and introducing the original mathematical model of the function of "good impression" on the new product;
- Correlation within a firm to launch budget process with "good impression" that consumers have about the new product and the function that controls the level of investment during this process, and moments when you need to still invest in promotion However within the proposed time;
- Highlighting the significant parameters involved in the mathematical model developed, which can be properly defined and particular to each category of products. These parameters can be choose by the specialists for follow the new products launching process on the market.

This model is very useful for the companies because applying it in new product launching process they will know all the time how much can they invest and which will be the time for whole process during. This model approach is considered for the  $y(0)=0$ , which means that at initial moment  $t=0$  the company has no cost and this is very important.

As future research it could be very interesting to consider that the company will have some costs at the beginning of the launching process which means that it could be an optimal control problem with mobile extremities made of the rectangles.

#### References

- [1] C. Dobocan, "Theoretical researches and contribution regarding new products launching process on the market", PhD Thesis, Technical University of Cluj-Napoca, Romania, 2010.
- [2] M. Baxter, "Product-Design – A Practical Guide to systematic methods of a new product development", Chapman & Hall, UK, 1995.
- [3] I. Blebea, C. Dobocan, "Proiectarea Produselor – De la teorie la practică", [Product design – From theory to practice] UT Pres, ISBN 978-973-662-292-2, Cluj-Napoca, Romania, 2007.
- [4] I. Blebea, C. Dobocan, R. Morariu-Gligor, "Optimal control problem in new products launch- Optimal path using a single command", Mechanisms and Machine Science 5, New Trends in Mechanism Science, ISBN 978-90-481-9688-3 DOI 10.1007/978-90-481-9689-0 EUCOMES, Cluj-Napoca, sept, 2010, p. 467-473.
- [5] A. Buratto, L. Grosset, & B. Viscolani, "New products introduction: goodwill, time and advertising cost", 42, p. 1343-1347, 2006.
- [6] A. Buratto, B. Viscolani, "An optimal control student problem and a marketing counterpart", Mathematical and Computer Modelling, 20:19-33, University of Padova, Italy, 2002.