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# Modeling and Simulation of Hydraulic Actuated Multibody Systems by Bond Graphs

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## Abstract

This paper deals with modeling and simulation of multibody systems consisting of rigid and flexible bodies actuated by hydraulic motors. To develop the corresponding model bond graph technique is used that provides a systematic component oriented methodology. The proposed procedure is explained on the example of rotary crane. The simulation model was developed and simulation conducted using BondSim<sup>©</sup> program. The model developed is based on physical modeling philosophy by systematically decomposing it into the components and using the models from the program library. Mathematical model of the system may be machine generated in the form of differential algebraic equations (DAEs) and solved using a solver capable of solving such models up to semi implicit index two models.

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## 1. Introduction

Rotary cranes are widely used for transport and handling of large loads at ports, construction work and industry. From economical point of view every effort has been made to design lighter and stronger crane constructions having large payloads. In addition, in many applications the load transfer has to be carried out as fast as possible. All these requirements introduce many problems in operation of the cranes and distort their dynamic performances. To help solving such problems dynamic analysis of the cranes attracts the attention of many researchers.

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A review of available crane models, the applied control strategies, classifications of the applications and their limitations are presented in [4]. Simulation methods for determining the dynamic responses of the mobile cranes with hydraulic drives are presented in [3].

In this work bond graph approach is used for model development of hydraulic actuated multibody system. To verify proposed approach, the procedure is applied to modelling of a rotary crane from [3]. Causal bond graph models of basic hydraulic components are presented in [6,7]. Model of a spatially actuated telescopic rotary crane in framework of causal bond graph technique is developed in [9]. However, unlike to the method applied in [9], the causalities are not taken into consideration. That leads to DAE models. The model developed herein is a complex one consisting of flexible and rigid 3D components. The procedure used is based on component model approach of [1] using BondSim© visual modeling and simulation environment, that enables using components from the program library. We note also that BondSim© supports the causal bond graphs as well, but in this paper due to complexity of presented flexible crane model, such approach has not been applied.

## 2. Model development procedure

Generally, multibody systems are composed of the rigid and flexible bodies (trusses, beams, shells, plates, and plane, axisymmetric and 3D solids). Rotary crane, to which is applied the proposed modeling procedure is shown in Fig.1a. Its system level bond graph model is depicted in Fig.1b. The crane consists of rotary platform that rotates about  $Z_0$  axis of the global inertial coordinate frame, denoted by  $O_0X_0Y_0Z_0$ . Platform flexibility is neglected and it is represented by 3D rigid body bond graph model - **Rotary platform**, Fig.1b. The boom, ropes, suspension ties and counterweight are placed on the platform and rotate jointly with it. The boom is a long and slender component represented by **Boom** component model in Fig.1b, which takes into account its flexibility. It is developed as a slender beam represented as aggregation of the beam finite elements (FE).

Flexibility of the suspension ties and the ropes is also considered in this work. Only axial deformation of the suspension ties is significant and, thus, they are modeled by a truss element (component **Suspension ties** in Fig.1b). The ropes according to idea of [3] are developed as a spring element. Load and counterweight are point masses and are presented by components **Counterweight** and **Load** in Fig.1b. All friction phenomena are neglected.

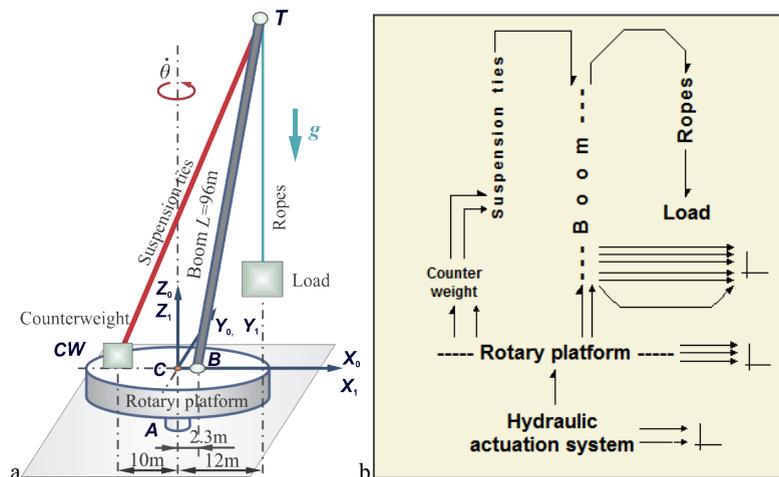


Fig. 1. Rotation crane: (a) Physical model; (b) Bond graph model.

### 2.1. Bond graph model of the platform

Rotary platform is considered as a constrained rigid body. Dynamic model of the platform is represented in the body frame  $O_1X_1Y_1Z_1$  rigidly attached to it. Procedure for modelling of rigid body is described in [1].

The constrained rigid body interacts with the connected bodies at the connection points. Thus, there is power flow from one body to the other, at e.g. points A and B (Fig.2a). Rotary platform of the crane is connected to the base at point A, to the boom at point B and to the counterweight at point CW (Fig.1a). Thus, its bond graph model has three power ports as shown in Fig.2b. We introduce two 6D flow and 6D effort vectors at these ports, consisting of the linear and the angular velocities components and the resultant force and moment components, respectively; all are defined in the body frame  $O_l X_l Y_l Z_l$  (All quantities defined in the body frame are denoted by superscript 1; ones defined in the global frame have superscript 0, which is omitted):

$$\mathbf{f}_k^1 = \begin{pmatrix} \mathbf{v}_k^1 \\ \boldsymbol{\omega}^1 \end{pmatrix}, \mathbf{e}_k^1 = \begin{pmatrix} \mathbf{F}_k^1 \\ \mathbf{M}_k^1 \end{pmatrix}, k = A, B, CW. \tag{1}$$

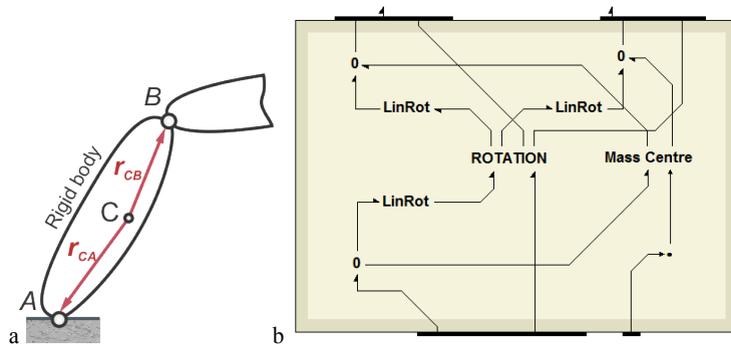


Fig. 2.(a) Constrained body; (b) Bond graph model of rigid body.

Initially, the body coordinate frame  $O_l X_l Y_l Z_l$  is coincident with the inertial frame  $O_0 X_0 Y_0 Z_0$ . During platform rotation by an angle  $\theta$  about axis  $Z_0$ , the orientation between these two frames changes and it is defined by the rotation matrix:

$$\mathbf{R}_1 = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{2}$$

Velocities at a port are related to the velocity of the body mass centre by:

$$\mathbf{f}_k^1 = \begin{pmatrix} \mathbf{I} & -\mathbf{r}_{Ck}^1 \times \\ \mathbf{0} & \mathbf{I} \end{pmatrix} \mathbf{f}_C^1, \quad k = A, B, CW, \tag{3}$$

where  $\mathbf{f}_C^1$  is 6D flow vector of the mass centre composed in the same way as port flows in Eq. (1) and  $\mathbf{r}_{Ck}^1$  is the vector of material coordinates at connection points ( $k=A,B,CW$ ) expressed in the body frame.

Bond graph model of the platform - **Rotary platform** - is shown in Fig.2b. Components **0** represent the arrays of flow junctions that describes the port velocities relationship given by the first three rows of Eq.(3). Components **LinRot** evaluate transformations between linear and angular quantities, defined by the skew-symmetric tensor operations in Eq.(3). The linear velocity at the port is sum of velocity of mass centre and the relative velocity of the port due to rotation of the body about the mass centre. On the other hand the angular velocity components do not change across the junctions.

If  $m$  is the body mass and  $\mathbf{F}_C$  is the resultant of the forces reduced to the mass centre, the translational part of the

motion is described in the global frame by:

$$\mathbf{p} = m\mathbf{I}\mathbf{v}_C, \frac{d\mathbf{p}}{dt} = \mathbf{F}_C. \quad (4)$$

The velocity of the mass centre may be represented in the global frame by:

$$\mathbf{v}_C = \mathbf{R}_1 \mathbf{v}_C^1. \quad (5)$$

The translational part of the platform motion is represented by component **Mass Centre**.

The rotation of the platform in the space is described by component **ROTATION**, which represents famous Euler equation of the body rotation, and which states that the global rate of change of the moment of momentum is equal to the corresponding local change and the part convected by the body rotation:

$$\frac{d\mathbf{H}^1}{dt} + \boldsymbol{\omega}^1 \times \mathbf{H}^1 = \mathbf{M}^1. \quad (6)$$

## 2.2. Bond graph model of the boom

The boom is a long and slender and its model is developed as collection of 3D beam finite elements [8] connected back to back. The component level bond graph model of the beam FE is shown in Fig.3. Its ports are defined, similarly to bond graph model of the rigid body component by 6D flow and effort vectors, Eq.(1), but now defined in the global frame. Dynamics of large the rigid body motion is defined in the global frame by component **Rigid Body Motion** that makes the mass matrix relatively simple. The modelling approach used is based on co-rotation formulation in which a co-rotation frame plays a fundamental rule. It is defined in such a way that the rigid body motion with respect to this frame is completely eliminated. Thus, the deformations of the finite element are represented in this frame using corresponding beam deformation theory (Euler, Timoshenko, or other). It is represented by component **Deformation** (in the middle of Fig.3). The other components provide the transformations between flow and effort vectors from one to the other coordinate frame, as well as other necessary coordinate transformations. Note that the large rotations of the FE are modelled by Rodrigues formula.

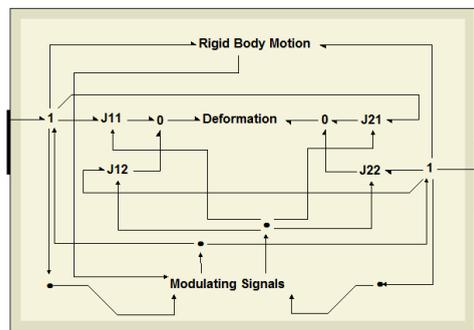


Fig.3. Bond graph model of 3D beam finite element.

## 2.3. Hydraulic actuation system

A simple hydraulic closed actuation system is used as a drive system of the crane. Fundamentals of hydraulic control systems are given in [5]. Nonlinear control problem of electro-hydraulic system is considered in [10].

A PID controller controls a variable displacement pump according to law:

$$x(t) = K_p \cdot err + K_d \frac{d(err)}{dt} + K_i \int (err) dt; \quad err = \dot{\theta}_r - \dot{\theta}, \quad (7)$$

where  $K_p, K_d, K_i$  are the parameters of controller, and  $\dot{\theta}$  and  $\theta$  are angular velocity and displacement of the rotary platform (index  $r$  means referent value of these quantities).

The pump supplies the hydraulic motor with high pressure hydraulic oil that via gear unit drives the platform (Fig. 4). Model of hydraulic motor is realized in form of causal bond graphs as given in [6].

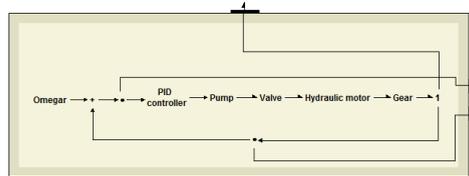


Fig. 4. Hydraulic actuation system.

### 3. Numerical example

To verify proposed approach the crane from [3] is taken as an example. The platform of the flexible crane rotates about  $Z_0$  axis with angular velocity that changes according to trapezoidal profile, defined by:

$$\omega_r = \dot{\theta}_r = \begin{cases} 0.25 \cdot t & t \leq 2 \\ 0.5 & 2 < t < 10 \\ -0.25 \cdot (t - 12) & 10 \leq t \leq 12 \\ 0 & t > 12 \end{cases} [rad / s]. \quad (8)$$

A part of geometrical parameters of the crane is already presented in Fig.1a. The other geometrical and the material parameters are given in Table 1.

Table 1. The crane parameters.

Parameters	Value
The modulus of elasticity [Pa],	1.08e11
The load mass with hook [kg]	60.526
The platform mass [kg]	53
The mass of counterweight [kg]	150
The boom moment inertia of cross-section [m <sup>4</sup> ]	0.0836 and 0.0665
The cross-section area of the boom [m <sup>2</sup> ]	0.0443
The cross-section area of suspension ties [m <sup>2</sup> ]	0.0084
The cross-section area of the ropes [m <sup>2</sup> ]	0.0004
The length of the ropes [m]	71.51

The simulation has been run using the time step of 0.001 s and absolute and relative error tolerances of 1e-6. Values of gains of PID controller are  $K_p=5, K_d=50$  and  $K_i=2$ .

Simulation results are presented in Figs.5a-e. The boom tip follows circular trajectory with radius of 12 m in  $X_0Y_0$  plane, as shown in Fig.5a. Transversal displacements of the boom tip are presented in Figs.5b and 5c. The obtained

angular displacement and velocity, as well as the error in angular velocity and acceleration are shown in Figs.5d,e and f. The results presented are in a good agreement with ones presented in [6]. Simulation statistical data, automatically generated by BondSim©, are given in Table 2.

Table 2. Simulation parameters presented by BondSim©.

The overall size of the model:	
The total number of the equations:	3544
The total number of the partial derivative matrix entries:	11303
The total number of the time rate variables matrix entries:	438

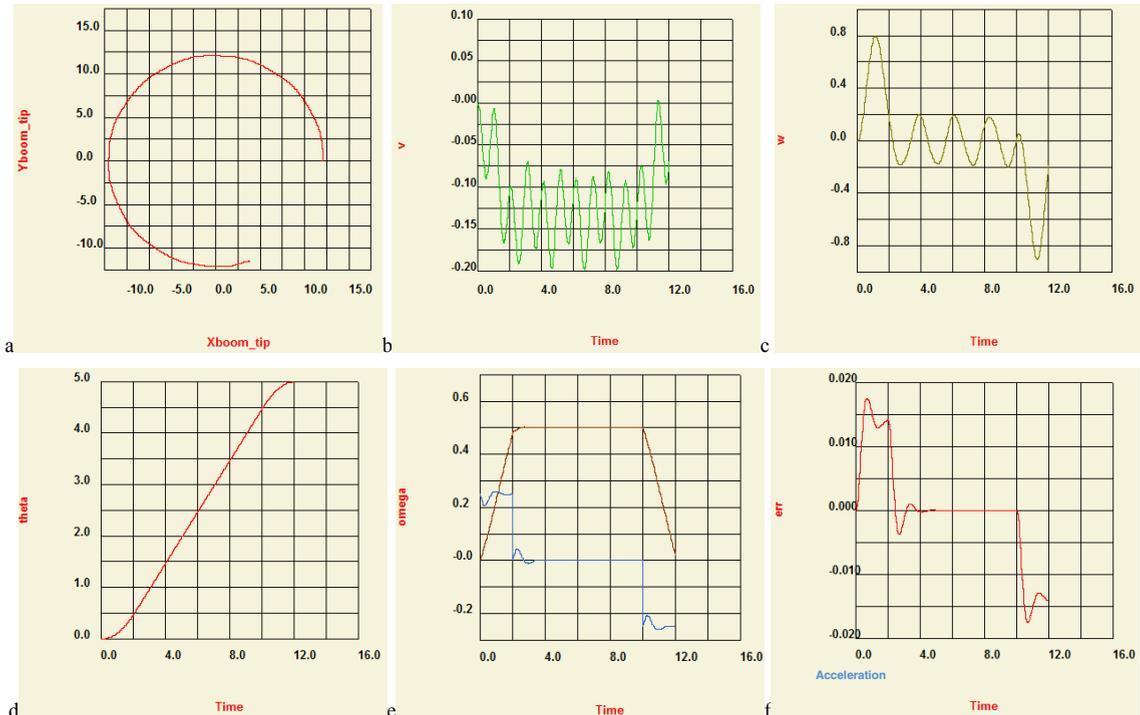


Fig. 5. Simulation results: (a) Trajectory of the boom tip in the global frame;(b) Transversal deflection; (c) Transversal deflection; (d) Obtained the rotation angle;(e) Obtained the rotation angular velocity (f) Obtained angular velocity and acceleration errors.

#### 4. Conclusion

A model of hydraulic actuation rotary crane is developed in this paper. The crane consists of a rotary platform, represented as a rigid body, the boom, modeled as the aggregation of beam finite elements, the suspension ties modeled as a truss element and ropes represented by a spring. Unlike to other approaches, the causalities, well known in bond graph theory, are not taken into consideration. Due to complexity of model, an acausal bond graph technique is applied herein leading to DAE models that are solved using modified backward differentiation formulae. Proposed approach is based on systematic component model decomposition providing development of very complex system on more hierarchical levels. Also, component model approach supports model development reusing component from the program library. Accuracy and efficiency of proposed procedure is verified by comparison of the obtained simulation results with the results reported in the references and which shows a good agreement with them.

More attention in the future work will be paid to modeling of the hydraulic drive system in detail. Developed model of the rotary crane can serve as platform for future investigation, for example comparison different control methods.

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