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*Make Harmony between Technology and Nature, and Your Mind will Fly Free as a Bird*
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# MODELING OF HIGH-PRESSURE FUEL INJECTION SYSTEMS

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***Abstract**: The aim of this study is to present a mathematical model of the wave pressure in the Diesel injection system. The injection process of a high-speed Diesel engine was studied in detail, using an original computer program developed in MATLAB. The governing equations are solved by the use of the finite difference method with central pattern at space coordinate in combination with the separation of flux vector. Simulations show satisfactory results, but improvements are possible. Since the models are developed for certain conditions it was not expected to be valid for all working conditions.*
***Keywords:** injection, nozzle, finite difference, fuel pipe, mathematical model, simulation, partial diferential equation*

## 1. INTRODUCTION

Due to its high thermal efficiency, Diesel engines were extensively used for various purposes, from vehicles to power plants. Due to the pollution of the environment and the shortage of energy sources, many Diesel engine researches develop studies dedicated to the improvement of the performances and reduction of the toxic emissions [1].

Many of these researches are focused on the optimization of the fuel injection system. The fuel injection system affects the Diesel engine combustion process through the control of injection time, injection quantity and injection rate [2].

Many simulations of the Diesel engine fuel injection system are carried out in order to understand the fuel injection system optimum parameters [3]. The method of characteristics and the finite difference method have been used to solve the governing equations. Recent studies show that finite difference method is superior to the method of characteristics concerning computation time and nonlinearity [4].

## 2. SIMULATION

The task of the fuel injection system is to meter the appropriate quantity of fuel for engine speed and load, and to inject that fuel at an appropriate time. In order to model the entire injection system there must be taken into account all the components: pump, pipe and nozzle.

When forming equations of continuity and motion the following assumptions are considered:

1. All the equations have 1D spatial resolution.
2. Temperature change due to pressure and time during the cycle is not considered.
3. The vapour pressure of the fuel is small compared to the level of the pressure injection system and therefore it is assumed that cavitation will not occur.

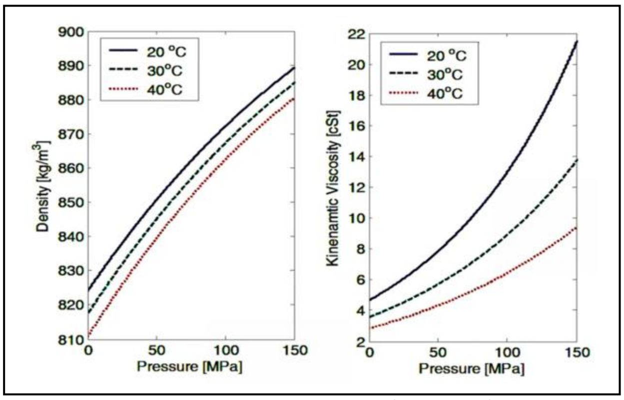


Fig. 1. Density and kinematic viscosity function of pressure and temperature

4. Elastic deformation in the injection system is not considered.

In a fuel injection system, pressure and temperatures can vary in a wide range of values, which leads to a wide variety of important fuel proprieties such as density, and kinematic viscosity. Having good fluid propriety models over a large range of pressure and temperatures is therefore essential for accurate modeling. Fig. 1 gives an overview of the Diesel fuel properties [5].

For this simulation the constant temperature assumption is considered, in these conditions the pressure equation being:

$$p = \beta_0(p, T_0)\frac{1}{\rho}\frac{(\dot{m}_{in} - \dot{m}_{out})}{V} \tag{1}$$

where $\beta_0$ bulk modulus at $T_0$ constant for the giving simulation, $\dot{m}_{in}$ and $\dot{m}_{out}$ are respectively the ingoing and outgoing mass flow rate of the volume $V$.

All elements can be connected by fuel line. In these hydraulic lines, pressure wave dynamics is taken into account. The motion equation of a hydraulic line is:

$$\frac{dq}{dt} - \frac{A}{\rho}\frac{dp}{dx} + u\frac{dq}{dx} + A \cdot g \cdot \sin(\alpha) + h(q) = 0 \tag{2}$$

with $q$ the volumetric flow rate, $\alpha$ the angle between the axis of the pipe with the horizontal and $h(q)$ the viscous friction term which is dependent on the relative roughness of the pipe wall.

The continuity equation is:

$$\frac{dp}{dt} + u\frac{dp}{dx} + \frac{\rho c^2}{A}\frac{dq}{dx} = 0 \tag{3}$$

where $c$ is the speed of sound.

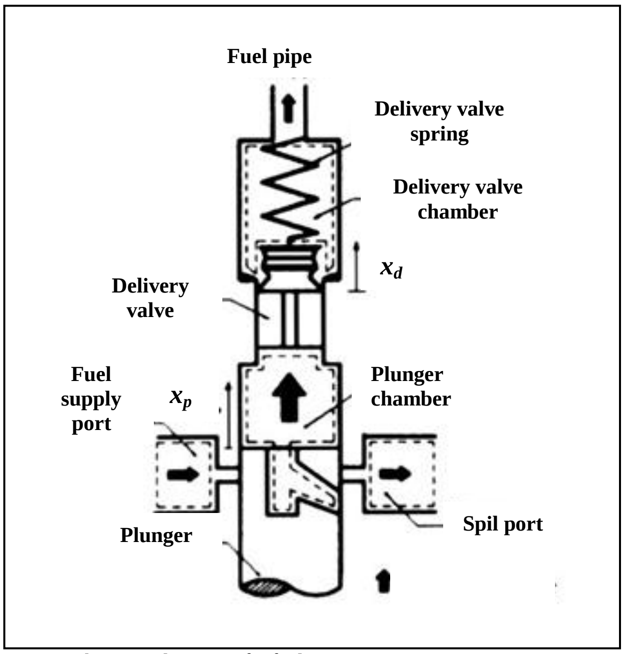


Fig. 2. Schematic diagram of a fuel pump

## 2.1 Fuel injection pump

The volume of the pump component can be divided into 3 control volumes such as plunger chamber, delivery chamber, and spill port as shown in Fig. 2. The continuity of flow in three control volumes and the equilibrium of forces on the delivery valve offer three governing equations.

An orifice model is used as a flow model for the interfacing control volume:

$$A_p \frac{dx_p}{dt} = \frac{V_p}{\beta_0}\frac{dp_p}{dt} + A_v \frac{dx_d}{dt} + Q_{pd} + Q_{ps} \tag{4}$$

$$A_v \frac{dx_d}{dt} + Q_{pd} = \frac{V_p}{\beta_0}\frac{dp_p}{dt} + \frac{dw_1}{dt} \tag{5}$$

$$M_d \frac{d^2 x_p}{dt^2} + k_d x_d = A_v (p_p - p_d) - f_d \tag{6}$$

where: $A_p$ plunger cross section area, $x_p$ plunger movement, $V_p$ plunger chamber volume, $p_p$ plunger chamber pressure, $A_v$ delivery valve area, $x_d$ delivery valve movement, $Q_{pd}$ flow between plunger chamber and delivery chamber, $Q_{ps}$ flow between plunger chamber and spill port, $w_1$ cumulative flow of first pipe node, $M_d$ delivery valve moving mass, $k_d$ delivery valve spring constant, $p_d$ delivery valve chamber pressure and $f_d$ delivery valve initial spring force.

## 2.2 Fuel injection pipe

The continuity equation, momentum equation and the equation of state (1) are taken into account to calculate the flow and the pressure in the fuel injection pipe. The momentum equation can be expressed by (7) in the form of a cumulative flow equation.

$$\frac{d^2 w}{dt^2} = -\frac{A_{pipe}}{\rho}\frac{\partial p}{\partial x} - \frac{32\nu}{D^2}\frac{\partial w}{\partial t} \tag{7}$$

where $A_{pipe}$ is the area of fuel injection pipe, $\nu$ is the kinematic viscosity, $D$ fuel injection pipe inner diameter

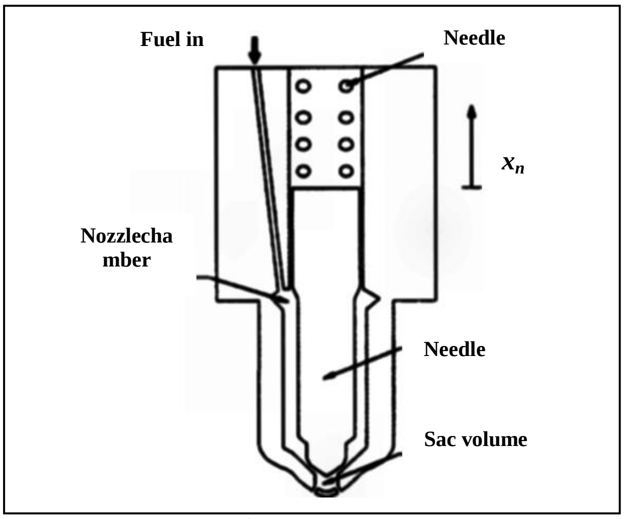


Fig. 3. Schematic diagram of a nozzle

## 2.3 Fuel injection nozzle

The fuel injection nozzle volume can be separated into two control volumes, i. e. nozzle chamber volume and sac volume as presented in figure 2. In the deriving of the governing equations it is assumed that the injection rate into the combustion chamber is the same as the fuel flow rate from the nozzle chamber to the sac volume. By considering the continuity equation and the equilibrium of force on the needle valve, (8) and (9) can be obtained.

$$\frac{dw_l}{dt} = \frac{V_n}{\beta_0}\frac{dp_n}{dt} + A_n \frac{dx_n}{dt} + C_d A_{nseff}\sqrt{\frac{2(p_n - p_c)}{\rho}} \tag{8}$$

$$M_n \frac{d^2 x_n}{dt^2} + k_n x_n = (A_n - A_{sc})p_n + A_{sc} p_s - f_n \tag{9}$$

where $A_{nseff} = \frac{A_{hole} A_{no}}{\sqrt{A_{hole}^2 + A_{no}^2}}$

and $w_l$ cumulative flow of last pipe node, $V_n$ nozzle chamber volume, $p_n$ nozzle chamber pressure, $A_n$ nozzle cross section area, $x_n$ needle lift, $C_d$ discharge coefficient, $A_{nseff}$ effective flow area from nozzle chamber to cylinder, $p_c$ cylinder pressure, $M_n$ needle moving mass, $k_n$ needle spring constant, $A_{sc}$ sac volume cross section area, $p_s$ sac volume pressure, $f_n$ needle initial spring force, $A_{hole}$ nozzle hole area, $A_{no}$ flow area from nozzle chamber to sac volume.

## 2.4 Boundary conditions

Fuel is delivered into the cylinder through the injection pump, the injection pipe, and the injection nozzle. Fuel is compressed by the plunger lift in the pump and injected into the cylinder. For that reason, the plunger lift and the cylinder pressure are used as boundary conditions. The plunger lift is calculated from the cam profile and roller's diameter in the in-line pump. The cylinder pressure is measured by the pressure transducer mounted in the cylinder head. As mentioned above, the entire fuel injection system is modeled as a three-component model i. e. pump, pipe, and nozzle. In order to calculate the whole system, it is assumed that the delivery chamber pressure is identical to the first node pressure of the injection pipe. Similar assumption is also applied to another boundary section where the last node of the fuel pipe meets the nozzle chamber.

## 2.5 Calculation method

When equations of the model are discretized by the use of the finite difference method, they are nonlinear equations. The fuel injection pipe governing equations are turned into a finite difference form by the Leap-Frog scheme. This explicit scheme converges when (10) is satisfied, [6].

$$\frac{\Delta x}{\Delta t} \geq c = \sqrt{\frac{\beta_0}{\rho}} \tag{10}$$

For solving the system of partial differential equations which describes the hydrodynamical process of the flow in the high pressure line, we will use the method of finite differences with central pattern at space coordinate in combination with the separation of flux vector [6]. We will solve the one-dimensional equations systems in the following form:

$$\frac{\partial}{\Delta t}\{w\}+\frac{\partial}{\partial x}\{E\}=0 \tag{11}$$

i.e. the quasi-linear form:

$$\frac{\partial}{\Delta t}\{w\}+[J(w)]\frac{\partial}{\partial x}\{w\}=0 \tag{12}$$

where $\{E\}$ is the flux vector and $[J(v)]$ Jacobian, obtained by derivation of vector $\{E\}$ from vector $\{v\}$. Only the central pattern for approximation of the derivative along $x$-axis gives stable calculation for positive and negative pressure waves. The application of nonsymmetrical operators can increase stability, reduce the problems to the two-diagonal system of equations instead of the tree-diagonal one in implicit formulations, and provide better dispersion and dissipation characteristics. By approximating the partial derivative in space by a backward pattern, it can be concluded that asymmetrical approximation of the derivative cannot be constructed at space coordinate which will be simultaneously stable for both its positive and negative eigenvalues.

According to Euler theorem for homogeneous function, it follows that:

$$E=[J]\{w\} \tag{13}$$

As far as vector $\{E\}$ meets the necessary level of homogeneity and as far as $[J]$ has the appropriate number of linearly independent vectors, vector $\{E\}$ can be split in two parts, each suitable for its eigenvectors. One part will correspond only to positive own values, and the other to the negative ones, i.e.:

$$\{E\}=\{E^+\}+\{E^-\} \tag{14}$$

where $\{E^+\}$ corresponds to positive eigenvectors of the matrix $[J]$ and $\{E^-\}$ to negative eigenvectors of the matrix $[J]$. The equation (14) becomes:

$$\{E^+\}+\{E^-\}=([J^+]+[J^-])\{w\}=[J^+]\{w\}+[J^-]\{w\} \tag{15}$$

The computer code is written in the MATLAB environment using the built-in functinf of the platform and the software instruments developed in [6].

| Characteristics | Technical specification |
|---|---|
| Introduction type | Natural aspirated |
| Cylinder bore | 100 mm |
| Piston stroke | 175 mm |
| Compression ratio | 17.5 |
| Maximum torque | 250 Nm at 1400 rpm |
| Maximum power | 48 kW at 2400 rpm |
| Pump type Bosch | In-line pump |
| Plunger diameter | 10.5 mm |
| Pre-stroke | 3.2 mm |
| Fuel pipe inner diameter | 2 mm |
| Nozzle type | Hole type |
| Nozzle hole number | 4 |
| Nozzle hole diameter | 0.24 mm |
| Nozzle hole angle | 90 deg |
| Opening fuel valve pressure | 215 bar |
| Pipe length | 320 mm |

Tab. 1. Specification of engine and fuel injection system

## 3. EXPERIMENT AND ANALYSIS

The experiment is carried out to measure the fuel consumption, in-cylinder pressure, the fuel injection pipe pressure near the injection valve and needle lift for several regimes of the working domain of a Diesel engine.

The experimental set-up includes a 4 stroke cycle 4 cylinder Diesel engine T684 (tab. 1) made by Tractorul Brasov Romania, a dynamometer and a data acquisition system. The system can measure in-cylinder pressures and the injection pipe pressure, near injection valve, together with the needle lift with a resolution of 0.2 deg crankshaft rotation, at the same time.

Figures 4 and 5 present the measured and simulated fuel pipe pressures for the engine speeds of 1440 rpm and 2400 rpm at full load. The fuel pipe pressure of the simulation is similar to that of the measurement with slight differences.

The measured fuel pipe pressures and the simulated fuel pipe pressures matched well to each other except for the later stage of the injection when cavitation might be occurring, phenomena with is not taken into account.

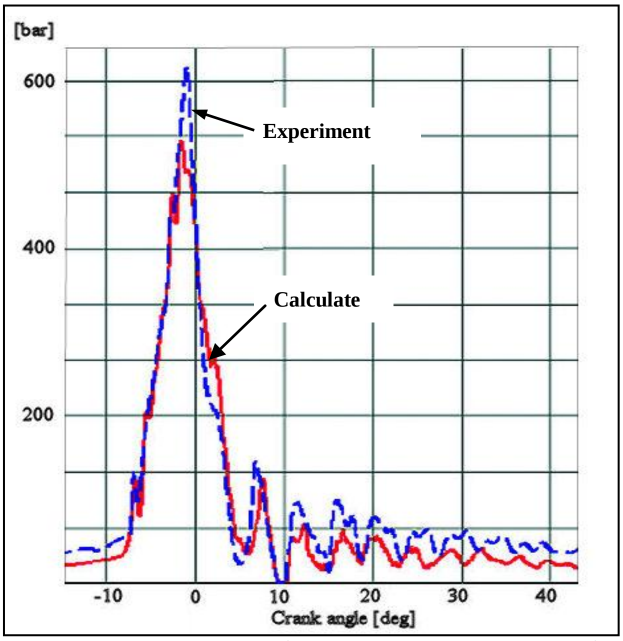


Fig. 4. Measured and simulated fuel pipe pressure at 1400 rpm and full load

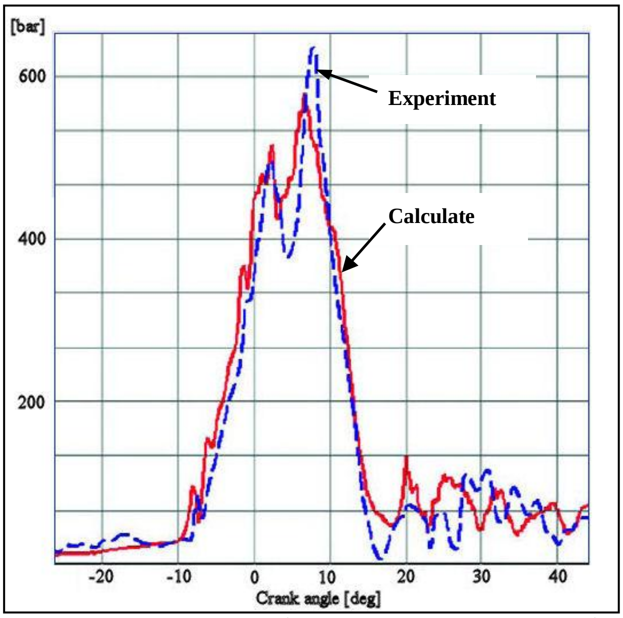

Fig. 5. Measured and simulated fuel pipe pressure at 2400 rpm and full load

The simulations for the working conditions corresponding to the maximum torque (1400 rmp) are in close agreement with the measurements (fig. 4), the measured and calculated data varying in concordance, but for the maximum power running conditions (2400 rpm fig. 5), a lag may be noticed. This delay can be explained by the assumption of system rigidity, that is not valid for high speed.

The results calculated with the program contain a number of error sources, such as:

1. The adoption of flow coefficient for the gap between the needle and nozzle is difficult and relatively arbitrary, since it could be selected from a wide range and for many conditions.
2. The injection mass per shot was measured from a running engine by the aid of the fuel mass flow rate. The injection mass was assumed to be equal in each cylinder. Therefore, the precise size of the injection mass of the measured cylinder is not known.
3. The properties of the fuel were not measured, being used several sources from the technical literature. This factor may cause errors of the calculated results.

The numerical method of finite differences with separation of the flux vector can be easily applied in solving the partial equations which describe the hydrodynamical processes in the high pressure line of the fuel injection system.

Selection of the integration step along both the time and space coordinates must satisfy the condition regarding the stable solution (10), but this is difficult for high speed and increases very much the computer code running time. Also, this integration time step must be compatible with time step used in integration of differential equations for boundary conditions in the delivery valve and the injector.

The advantage of splitting the vector *{E}* into two parts is in avoiding the transformation of the equations at each point in the calculation domain into a series of disconnected partial differential equations. Instead of transforming the equations each time, only vector's *{E}* components are calculated.

## 4. SUMMARY AND CONCLUSION

The study proposes and validates a mathematical model and a computer code which is capable to predict the pressures in injection system. The following conclusions are derived from this study.

1. The code offers good results in principal for low speed working conditions.
2. For high speed working conditions it is important to take into account cavitation and the elasticity of the components.
3. For calibration of the model, a very accurate measured data is need.
4. Numerical method needs improvements in order to increase the computing speed and the accuracy.
5. Simulation of flow in the nozzle needs 3D resolution for accurate prediction.

Targets for further research related to the present work is to improve the model attaching submodels for cavitation and elasticity of the components.

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