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ESTIMATION OF COMPLEX DERIVATIVES AND APPLICATION FOR FAULT DIAGNOSIS

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Abstract: *Using parameters of vibration (vibration displacement,vibration velocity and vibration acceleration) is not efficient for small and high speeds of shaft rotation, when there are no shock loads or frequency vibration is very high. When analyzing vibration it is effective to use the intensity of vibration acceleration changes and its derivatives. Order of derivative can be any real number. If the frequency of the vibration is not constant, then fractional derivatives are used. The intensity of the vibration acceleration change can be estimated by frequency spectrum of the vibration power.*
Keywords: *higher order complex derivatives, vibration signal, fault diagnosis, fractional order derivatives*

1. INTRODUCTION

Using parameters of vibration (vibration displacement,vibration velocity and vibration acceleration) is not efficient for small and high speeds of shaft rotation, when there are no shock loads or frequency vibration is very high. When analyzing vibration it is effective to use the intensity of vibration acceleration changes and its derivatives.

Order of derivative can be any real number. If the frequency of the vibration is not constant, then fractional derivatives are used. The intensity of the vibration acceleration change can be estimated by frequency spectrum of the vibration power. Lahdelma and his co-workers [Lahdelma, and Kotila, 2005] and [Lahdelma and Juuso, 2007] have introduced *complex* derivatives of vibration signal- for reliable condition measuring of rotating equipment.

It is nit easy to estimate derivatives of vibration signals. To estimate higher order complex derivatives of vibration signal a method based on differential algebra is proposed here.

The idea of numerical differentiation of signals using differential algebra tools [K. Yosida, 1984] has been proposed by Fliess and Mboup [M. Fliess *and al.*, 2006] and [M. Mboup *and al.*, 2009]. This method is exact and fast. Several applications have been reported for the signal processing, see for example [Villagrá, 2012]. We demonstrate the possibilities offered by this differential algebra method to enhance the information from noisy vibration signal. We show that it is possible to use the (standard) Operational Calculus methods to estimate the derivatives of fractional order of very noisy and fast signals.

2. ESTIMATION OF COMPLEX DERIVATIVES

If to determine frequency, at which frequency power density of the high-frequency vibration achieves a maximum,then coefficients of frequencies rate will be estimated with sufficient accuracy. The frequency of power spectrum depends on frequency of a spectrum of vibration under the following formula

$$L(f)=\frac{1}{n^2}\left|\sum_{k=0}^{n-1}u_k e^{-i2\pi f t_k}\right|^2 \quad (1)$$

Where
n- number of discrete measuring of estimations of signal of the vibration. After some simplifications the estimation of a power spectrum is determined by the formula

$$L(f)=\frac{1}{n^2}\left|\sum_{k=0}^{n-1}u_k\cos 2\pi f t_k - i\sum_{k=0}^{n-1}u_k \sin 2\pi f t_k\right|^2 \quad (2)$$

Now it is easy to determine frequencies corresponding to the max value of power spectrum. By the following equations the frequencies corresponding to the max value of a power spectrum can be determined

$$\begin{aligned}
&L(f)=\frac{1}{n^2}(u_0^2+u_1^2+\ldots+u_{n-1}^2)+\\
&+\frac{1}{n^2}(2u_0u_1+2u_1u_2+\ldots+2u_{n-2}u_{n-1})\times\\
&\times\cos(2\pi f dt)+\frac{1}{n^2}\times\\
&\times(2u_0u_2+2u_1u_3+\ldots+2u_{n-3}u_{n-1})\times\\
&\times\cos(2\pi f\,2dt)+\frac{1}{n^2}\times\\
&\times(2u_0u_3+2u_1u_4+\ldots+2u_{n-4}u_{n-1})\times\\
&\times\cos(2\pi f\,3dt)+\frac{1}{n^2}\times\\
&\times(2u_0u_{n-1})\cos(2\pi f(n-1)dt)
\end{aligned} \quad (3)$$

Let us assume that the periodic signal has the Fourier series expansion:

$$L(f) = U_0 + \sum_{k=1}^{n-1} U_k \cos \omega k dt \tag{4}$$

Where U_k denotes the amplitude of the kth harmonic, ω is the fundamental angular frequency.

Now it is easy to determine frequencies corresponding to the max value of power spectrum. By the following equations the frequencies corresponding to the max value of a power spectrum can be determined:

$$\begin{aligned} &-\sum_{k=0}^{n-1} U_k \cos 2\pi f t_k \sum_{k=0}^{n-1} U_k t_k \sin 2\pi f t_k + \\ &+\sum_{k=0}^{n-1} U_k \sin 2\pi f t_k \sum_{k=0}^{n-1} U_k t_k \cos 2\pi f t_k = L'(f) \end{aligned} \tag{5}$$

Under formula (5) the ergodic casual process proselected by a final set of material values $U_k = U(t_k)$ is investigated. We can use estimation of power to define harmonic with frequency f, this frequency qives the greatest contribution to Fourier series disintegration. For the lines composed from five pair of numbers (n-1=5) of measured physical magnitudes, formula (5) looks like

$$\begin{aligned} &- \cdot U_1 2\pi dt \sin(2\pi f dt) - \\ &- U_2 4\pi dt \sin(4\pi f dt) - U_3 6\pi dt \sin(6\pi f dt) - \\ &- U_4 8\pi dt \sin(8\pi f dt) - U_5 10\pi f dt \sin(10\pi f dt) = L'(f) \end{aligned} \tag{6}$$

Augmenting realization of investigated process up to a member $n-1$, formula (6) looks like:

$$\begin{aligned} &\sum_{k=1}^{n-1} 2k \cdot (U_0 U_1 + U_1 U_2 + \dots + U_{n-1-k} U_{n-1}) \times \\ &\times \sin(2k\pi \ f dt) = L'(f) \end{aligned} \tag{7}$$

Having constructed Shuster Periodogramm, it is possible to determine extreme frequency with the big fidelity, on which one the true spike of the response of fault is observed. Equation (7) represented in the real plane.

The nonlinear equations it is easier solve in the complex plane. To do this, let us jump to the complex arguments and write equation (7) dividing it by the imaginary unit.

$$\begin{aligned} &\frac{L'(f)}{i} = -\frac{2\pi dt U_1}{i} \sin(2\pi f dt) - \\ &-\frac{4\pi dt U_2}{i} \sin(4\pi f dt) - \dots \\ &-\frac{2\pi(n-1) dt U_{n-1}}{i} \sin(2\pi f (n-1) dt) \end{aligned} \tag{8}$$

Let us sum two equations:

$$\begin{aligned} &\frac{L'(f)}{i} + L'(f) = -2\pi dt U_1 [1-i] \sin(2\pi f dt) - \dots - \\ &- 2\pi(n-1) dt U_{n-1} [1-i] \sin(2\pi f (n-1) dt) \end{aligned} \tag{9}$$

We use Euler's formula:

$$\begin{aligned} &\frac{L'(f)}{i} + L'(f) = \\ &= -2\pi dt U_1 [1-i] \frac{e^{i(2\pi f dt)} - e^{-i(2\pi f dt)}}{2i} - \dots - \\ &- 2\pi(n-1) dt U_{n-1} [1-i] \times \\ &\times \frac{e^{i(2\pi f (n-1) dt)} - e^{-i(2\pi f (n-1) dt)}}{2i} \end{aligned} \tag{10}$$

After simple transformations we obtain the following expression:

$$\begin{aligned} &\frac{L'(f)}{i} + L'(f) = \\ &= \pi dt U_1 \left(\frac{e^{i(2\pi f dt + \frac{\pi}{4})} - e^{-i(2\pi f dt - \frac{\pi}{4})}}{1}\right) + \dots + \\ &+ \pi(n-1) dt U_{n-1} \times \\ &\times \frac{e^{i(2\pi f (n-1) dt + \frac{\pi}{4})} - e^{-i(2\pi f (n-1) dt - \frac{\pi}{4})}}{1} \end{aligned} \tag{11}$$

Now we can introduce the complex variable:

$$e^{i\omega dt} = z; \quad (e^{i\omega dt})^k = z^k \tag{12}$$

$$\begin{aligned} &L'(z) = \pi dt U_1 (e^{i\frac{\pi}{4}} z - e^{i\frac{\pi}{4}} \frac{1}{z}) + \\ &+ \dots \pi(n-1) dt U_{n-1} (e^{i\frac{\pi}{4}} z^{n-1} - e^{i\frac{\pi}{4}} \frac{1}{z^{n-1}}) \end{aligned} \tag{13}$$

The use of complex order derivatives increases the number of signal from which a signal with the best sensitivity is selected in order to detect the desired faults [Lahdelma and Juuso, 2007].

From the functions of a complex variable it is possible to take complex derivatives:

1) Type (N=3):

$$\begin{aligned} &L^{(3)}(z) = \pi dt e^{i\frac{\pi}{4}} \times \\ &\times \sum_{m=1}^{n-1} m U_m \left[m(m-1)(m-2) z^{m-3} + m(m+1)(m+2) \frac{1}{z^{m+3}} \right] \end{aligned} \tag{14}$$

Let us assume that

n-1=3

$$L^{(3)}(z) = \pi dt e^{i\frac{\pi}{4}} \times \left\{ \begin{aligned} &U_1 \left[1 \cdot 2 \cdot 3 \cdot \frac{1}{z^5} \right] + 2 \cdot U_2 \left[2 \cdot 3 \cdot 4 \cdot \frac{1}{z^5} \right] + \\ &+ 3 \cdot U_3 \left[3 \cdot 2 \cdot 1 \cdot z^0 + 3 \cdot 4 \cdot 5 \cdot \frac{1}{z^6} \right] \end{aligned} \right\}$$

$$L^{(3)}(f) = \pi dt \frac{\sqrt{2}}{2} \times \left\{ \begin{array}{l} 6 \cdot U_1 \cos(8\pi f dt) + 48 \cdot U_2 \cos(10\pi f dt) + \\ +18 \cdot U_3 + 180 \cdot U_3 \cos(12\pi f dt) + \\ +6 \cdot U_1 \sin(8\pi f dt) + 48 \cdot U_2 \sin(10\pi f dt) + \\ +180 \cdot U_3 \sin(12\pi f dt) \end{array} \right\} +$$

$$+ i\pi dt \frac{\sqrt{2}}{2} \times \left\{ \begin{array}{l} -6 \cdot U_1 \sin(8\pi f dt) - 48 \cdot U_2 \sin(10\pi f dt) + \\ +18 \cdot U_3 - 180 \cdot U_3 \sin(12\pi f dt) + \\ +6 \cdot U_1 \cos(8\pi f dt) + 48 \cdot U_2 \cos(10\pi f dt) + \\ +180 \cdot U_3 \cos(12\pi f dt) \end{array} \right\}$$

2) Type (N=5):

$$L^{(5)}(z) = \pi dt e^{i\frac{\pi}{4}} \times$$
$$\times \sum_{m=1}^{n-1} mU_m \left[\begin{array}{l} m(m-1)(m-2)(m-3)(m-4) z^{m-5} + \\ + m(m+1)(m+2)(m+3)(m+4) \frac{1}{z^{m+5}} \end{array} \right] \quad (15)$$

3) Type (N=7):

$$L^{(7)}(z) = \pi dt e^{i\frac{\pi}{4}} \times$$
$$\times \sum_{m=1}^{n-1} mU_m \times \left[\begin{array}{l} m(m-1)(m-2)\dots(m-6) z^{m-7} + \\ + m(m+1)(m+2)\dots(m+6) \frac{1}{z^{m+7}} \end{array} \right] \quad (16)$$

4) Higher Order Derivatives:

$$L^{(k)}(z) = \pi dt e^{i\frac{\pi}{4}} \times$$
$$\times \sum_{m=1}^{n-1} mU_m \left[\begin{array}{l} m(m-1)(m-2)\dots(m-k+1) z^{m-k} \pm \\ \pm m(m+1)(m+2)\dots(m+k-1) \frac{1}{z^{m+k}} \end{array} \right] \quad (17)$$

Let substituting the complex variable to the equation (17), we obtain the higher complex derivatives, depending on the frequency of vibration:

$$L^{(k)}(\omega) = \pi dt e^{i\frac{\pi}{4}} \times$$
$$\times \sum_{m=1}^{n-1} mU_m \left[\begin{array}{l} m(m-1)\dots(m-k+1)(e^{i\omega dt})^{m-k} \pm \\ \pm m(m+1)\dots(m+k-1) \frac{1}{(e^{i\omega dt})^{m+k}} \end{array} \right] \quad (18)$$

Where k is a complex number and $i=\sqrt{-1}$. The use of complex-order derivatives increases the number of signals from which a signal with the best sensitivity is selected in order to detect the desired faults.These are average values of sensitivity between features of the objects with known and unknown faultiest. The feature can be a signal value from C, B and F bearing of PT500 diagnostic device (fig.1, 2, 3).

Here, C bearing with damage to inner race (fundamental frequency is 117 Hz and rotation frequency 25 Hz) and B bearing with damage to outer race (fundamental frequency is 86 Hz and rotation frequency 25 Hz); bearing F with severe wear.

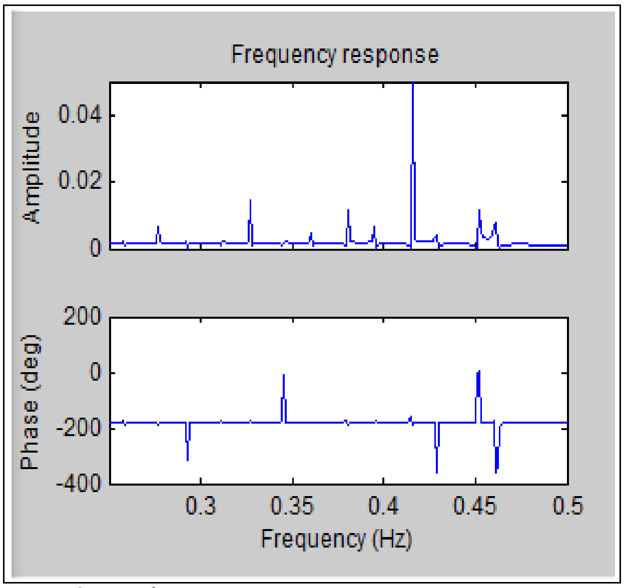

Fig.1. C bearing frequency response

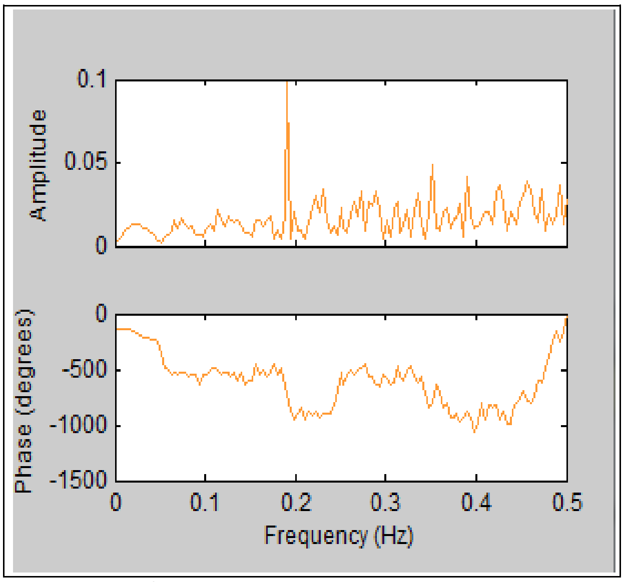

Fig.2. B bearing frequency response

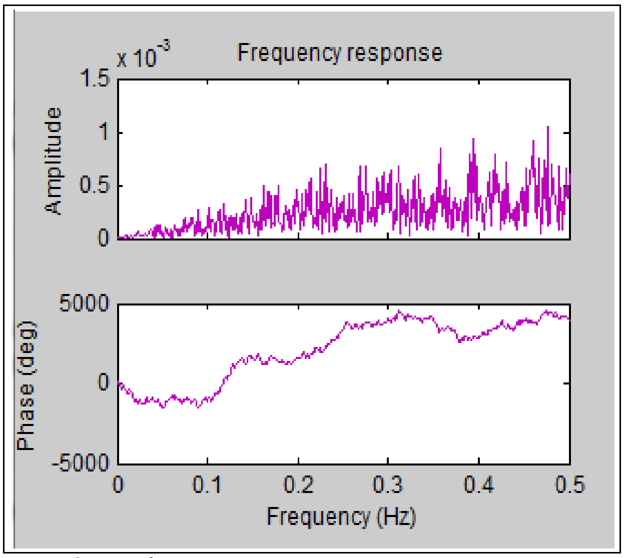

Fig.3. F bearing frequency response

The complex derivative can be examined at small rotation frequency.

3. CONCLUSION

Other multipliers of expression (18) are invariant to operation of differentiation. In the method of grids it is important to know the derivative values in nodes of interpolation. Methods from Operational Calculus allow to solve exactly the initial value problem. The calculation effort is mainly on integration. A suitable method of integration and of sampling period has to be selected. The first estimated values are very often far from the real values.

In the future we are going to develop a method of application of Operational Calculus for estimation of fractional order derivatives.

We apply the Operational Calculus to the problem of estimation of derivatives of signals as proposed in. [M. Mboup *and al.*, 2009].

The second main direction of the research is connected to the special method for detection of bearing defects. It consists of filtering of the bearing defect signal, removing most of the „noise" caused by structural vibration, misalignment etc., but retaining the useful information of the defect signal.

The result of the process enables users to analyze the signal and draw conclusions on the state of the bearing without interference noise from structural phenomenon in the machine.

The application of the complex order derivative to the acceleration signal is not restricting element. The received results can be applied to other types of signals as well, such as pressure signals.

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