Annals of DAAAM for 2012 & Proceedings of the 23rd International DAAAM Symposium, Volume 23, No.1, ISSN 2304-1382
ISBN 978-3-901509-91-9, CDROM version, Ed. B. Katalinic, Published by DAAAM International, Vienna, Austria, EU, 2012
Make Harmony between Technology and Nature, and Your Mind will Fly Free as a Bird
Annals & Proceedings of DAAAM International 2012

IDENTIFICATION OF PENDULUM OSCILLATION PARAMETERS USING MEMS ACCELEROMETER

SURIANSKY, J[ozef] & ADAMIK, M[iroslav]

Abstract: *The article deals with analysis of problematic, which is focused on identification of pendulum oscillation parameters. Theory is based on mathematical description of physical pendulum with real damping and kinematics theory. In the beginningthe article describes experimental laboratory model of physical pendulum with precise construction. Simultaneously it describes wireless sensor system with dual axis MEMS accelerometer, which is used for components accelerations measurement offree oscillatingpendulum arm. The article presents methodology for solution which is based on expression of boundary conditions and follows prediction of actual angle displacement in next period of oscillation. Experimental measurement and its subsequent evaluation determine properties and usefulness of principles of measurement and evaluation in purposes of control and visualization. Final evaluation offers suggestions for future improvement of described method.*
Keywords: *physical pendulum, damped oscillation, MEMS sensor, accelerometer, wireless sensor system*

1. INTRODUCTION

In many cases of engineering practice we encounter with the problem of loosely hanging body manipulation such as load of gantry or tower crane. Manipulation parts can be except the conventional materialalso container with molten metal used in metallurgical plant. In this case the manipulation has to be followed by precise technological procedure.

In the process of handling is the crane’s manipulation base (support) driven by motor acceleration. In pursuance of load inertia, angle displacement from basic position (vertical) has been created. This displacement has been used as an initiative for creation of tangential part of gravity acceleration, which keeps pendulum arm in permanent but damped swinging motion.So the amplitude of this motion is the main limiting factor for handling speed and its acceleration. And therefore measurement of pendulum oscillation amplitude (and of course frequency) and all kinematic parameters can be used to ensure quality handling. Handling quality requirements also includes conditions for compliance of technological procedure or prevention of collision and emergency state.

For measurement of angular displacement many relative principles of sensor operation has been created [1].In the case of deformation hinge (arm) they lost their utility and also completely preclude their technical use.Actual trend on the field of sensor devices are absolute MEMS sensors, whoseapplication on the oscillation object is possible to measureexpression of the drives thus accelerations.

We expect that such plane motion of pendulum can be completely measured by using dual axis accelerometer (planar movement). The problem of mathematical expression of sensed acceleration components lies on dilemma with separation of dynamic and static components of acting accelerations in the gravitational field of earth [2]. And therefore the article deals with interdisciplinary problem analysis of pendulum oscillation parameters. This approach however tolerates some degree of inaccuracy (human vision) in the context of dynamic processes visualization problematic (virtual reality).

2. LABORATORY MODEL OF PHYSICAL PENDULUM

For simulation of hanging load free oscillation, a model of physical pendulum has been built (Fig. 1). Massive steel construction allows except the free oscillation also possibility of constant amplitude oscillations maintenance by the disk motor and electronic control circuit. Precise deposition of rotary shaft provides that the all kinematic parameters of pendulum are planar only.

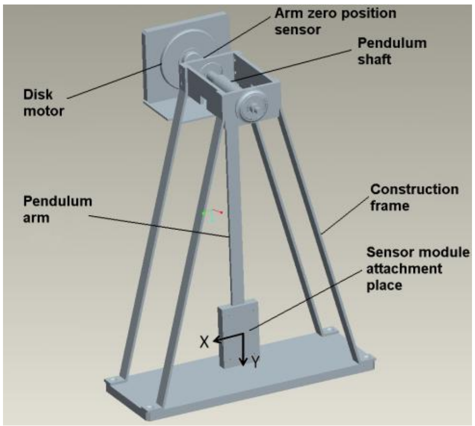

Fig. 1. Physical pendulum model construction

The end place of pendulum arm provides possibility of fixing a wireless sensor node with built-in dual axis MEMS accelerometer.

2.1 Wireless sensor system

For the purposes of remote measurements the transport wireless network has been created [3]. Wireless connection has been performed by the XBee communications modules of international radio standard IEEE 802.15.4. These modules were embedded to the circuit’s boards of sensor node (Fig. 2) and sink node.

The core of sensor node that has been constructed is the microcontroller MSP430, which is intended for acquisition data sequence control. Data is sensed and processed by the professional dual axis MEMS accelerometer ADIS16006 which communicates by the SPI serial bus.

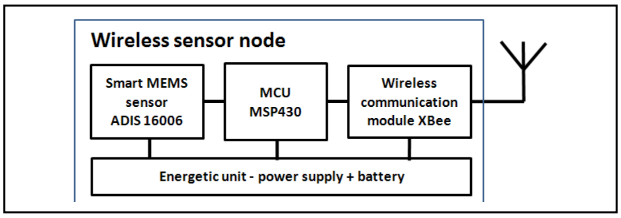

Fig. 2. Sensor module block structure

This sensor provides 12 bit resolution on the ±5g measuring range and 2250Hz limit frequency of output low-pass filter. Calibrated software conversion determines minimal sensitivity of $0.0412 m.s^{-2}$ per LSB. Basic sampling frequency of 104Hz of sensor node has been programmed. Acquired data is formed to structure in which is easy to check their integrity with the PC application.

The sensor has been fixed on the pendulum arm under the strict conditions. The X measurement axis of sensor must be in any time of motion identical with direction of tangential acceleration on the 42cm measurement radius (with circle trajectory). In the same way the Y measurement axis of accelerometer was fixed to the axis of the suspender in which centrifugal acceleration is concentrated.

3. PHYSICAL PENDULUM PROBLEMATICS

For the purpose of parameters calculation principle proposal is necessary to analyze theproblems from multiple scientific disciplines simultaneously. With following experiment the evaluation of designed principle properties can be performed.

3.1 Physical pendulum with real damping

Physical pendulum is a body, which is adapted for free rotation around the axis, which has been situated over its gravity center. In this case the body is able to act oscillation movement with influence of its weight gravity acceleration [4]. So the physical pendulum is a real object, whose movement is caused by tangential component of gravity.It is the computational equation (1). And simultaneously for angle $\varphi < 5°$ we can consider some simplifications.

$$F_t = -mg.sin\varphi = -mg.\sin(\omega t) \approx -mg\varphi \; [N] \qquad (1)$$

Because this component of gravity force caused a torque M, there is a possibility to express the pendulum equation of motion (2).

$$M = J\varepsilon = -dmg.\varphi \; [Nm] \qquad (2)$$

Where, $J$is the pendulum's moment of inertia in regard to rotational axis. Parameter d is the distance between axis of rotation and gravity center of the body. And finally ε is the angular acceleration of pendulum defined by (3).

$$\varepsilon = -\omega^2\varphi \; [rad.s^{-2}] \qquad (3)$$

With expression of ε value from the (2) and (3) equations main angular frequency (4) is possible to obtain.

$$\omega = \sqrt{\frac{dmg}{J}} \; [rad.s^{-1}] \qquad (4)$$

However the motion of the real pendulum will be dumped by the influence friction forces. The motion will be dumped until it will be stopped. Generally force F acts on the body, which is created by summary of inertia force F_i and overall friction force F_f. From the general analysis we can express the motion equation of damped oscillation in differential form (5).

$$\frac{d^2\varphi}{dt^2} + 2\delta\frac{d\varphi}{dt} + \omega^2\varphi = 0 \qquad (5)$$

Where, δ is the damping coefficient. Solution of this equation is the expression of damped oscillationangle displacement course by (6).

$$\varphi = \varphi_m.e^{-\delta t}.\sin(\omega_1 t) \; [rad] \qquad (6)$$

Where, φ_m is the oscillation amplitude and $\omega_1$is main angle frequency of damped pendulum by (7).

$$\omega_1 = \sqrt{\omega^2 - \delta^2} \; [rad.s^{-1}] \qquad (7)$$

In real conditions it is very difficult to identify weight of hanging load m. So the value computation of center position d and simultaneously frequency ω_1 is really impossible. Therefore the expression of damping coefficient $\delta$from measurement course of actual angle displacement φ is very useful.

3.2 Point motion kinematics on the circular trajectory

Each point of oscillating load moves on the circular trajectory on$r$ radius with regard to axis of rotation. In the context of circular trajectory point motion investigation the solution in the tangent – normal coordinate system is very useful. From the kinematic theory [5] the computational relations of remoteness, velocity and acceleration is well known.But in the case of pendulum we cannot forget the gravity acceleration components action. Gravity acceleration of g value has been divided to two perpendicular components, which depends on the actual angle position φ. Therefore this componentwill be added to the measurement axis and resultant equations of tangential (8) and normal (9) will be in next form.

$$a_t = r\varepsilon + g.sin\varphi \qquad (8)$$

$$a_n = r\omega^2 + g.cos\varphi \qquad (9)$$

These values of accelerations are directly measurable by the accelerometer and these computational relationships are valid in any time of pendulum movement.

3.3 Inertial measurement system

Inertial system is the autonomous system, which is capable to evaluate location, velocity and acceleration data (axial, angular), without necessity of using the external navigation information [6]. Measurement system uses inertial sensors such as accelerometer and gyroscope, which is principally based on 1st and 2ndNewton's law. Information of the location is evaluated from the initial location and continuous measurements of acceleration, which is thenintegrated in time.For the explicit determination of pendulum position the angle displacement φ from the zero (static) position φ_0 is sufficient. Because the angularvelocity of rotation is measured especially by the gyroscope, then the task of inertial computation will lie on the solutionof equation (10).

$$\varphi = \int \omega \, dt \qquad (10)$$

For thedescription of theω function between the two sensed samples (ideally $\Delta t \rightarrow 0$) is except the linear approximation function also possible to use the cosine function, which can cause complications in calculation (φ is unknown).

3.4 Boundary conditions solution

More important in respect of diagnosticthan the information of position, velocity and acceleration in any time of pendulum motion is period, amplitude and phase of oscillation. The value of actual angle of displacement can be directly identified only in two boundary conditions:

1. In the case, when the pendulum is passing through the zero position, i.e. when the $\varphi = 0$. In this moment the cosine compound of gravity acceleration can be eliminated completely. We can express the amplitude of angular velocity ω_m by the (9) to equation (11).

$$\omega_m = \sqrt{\frac{a_n - g}{r_m}}_{(\varphi=0)} \qquad (11)$$

   The calculation problem is transferred to searching of functional maximum value of $\omega$and alsoa_n functions.Simultaneously in this moment is the tangential acceleration value a_t= 0.
2. In the case, when the pendulum is passing through the extreme position, it means that $\varphi = \max = \varphi_m$ (amplitude of oscillation). The pendulum in this position is staying at the moment ($\omega = 0$). And then we can eliminate parameter of centrifugal accelerationfrom the equation (9). And then for the amplitude of angle displacement equation (12) applies. Simultaneously the tangential acceleration value reaches its local extreme($a_t = \pm$max).

$$\varphi_m = \arccos\left(\frac{a_n}{g}\right)_{(\omega=0)} \qquad (12)$$

It means that the algorithmic problem is based on searching for normal and tangent acceleration extremes. The direction of actual displacement ($\pm\varphi$) can be identified from the polarity of tangential acceleration. We establish the rule, when the negative polarity of a_t means anti-clockwise direction of pendulum deflection (right said, positive angle). It's valid analogous for left deflection of movement.

3.5 Pendulum displacement prediction

As we mentioned, the course of pendulum displacement follows sinusoidal shape which is influenced by damping (6). Therefore, if the value of actual displacement from the gyroscope (or inclinometer) isn't available, the course between two extreme position $\varphi \in (-\varphi_{max}, +\varphi_{max})$ is possible to predictive generation with using (6) equation. The precise prediction can start in the moment, when we quantify the main period of damped oscillation from two corresponding samples of displacement amplitude by (13).

$$\omega_1 = 2\pi \frac{1}{t_{\varphi m\,2} - t_{\varphi m\,1}} = 2\pi \frac{1}{T} \; [rad.s^{-1}] \qquad (13)$$

Simultaneously the damping coefficient δ, which equation has the following form (14).

$$\delta = \frac{\ln\left(1 - \frac{\varphi_{m1} - \varphi_{m2}}{\varphi_{m1}}\right)}{T} \; [s^{-1}] \qquad (14)$$

The damping coefficient and also the main angle oscillation frequency must be updated in the measurement process continuously. This makes it possible for best prediction of load pendulum position in the next time interval T up to next exact amplitude coming. The predicted angle allows that we can calculate all kinematic parameters of circle motion (φ, ω, ε) by the mentioned (8) (9) equations or other definitional relationship in any time of oscillation.

4.ANALYSYS OF FREE OSCILLATING PHYSICAL PENDULUM

The measurement on the physical pendulum model has been performed. The pendulum was set in motion by maximal right deflection and next free wobbled with gravity acceleration action.

4.1 Measured pendulum acceleration compounds

Acceleration compounds of damped pendulum oscillation were measured by early mentioned wireless sensor node and processed by software application which was especially created for this purpose. Measured data were modified and converted using calibration coefficients set for each axis to acceleration value. On this resultant data dual channel Kalman's filter was applied with experimentally defined value of process dynamic coefficient and predicted value of noise ratio. Process of measured tangential and normal accelerations of pendulum oscillations up to its stop is shown on next figure (Fig. 3).

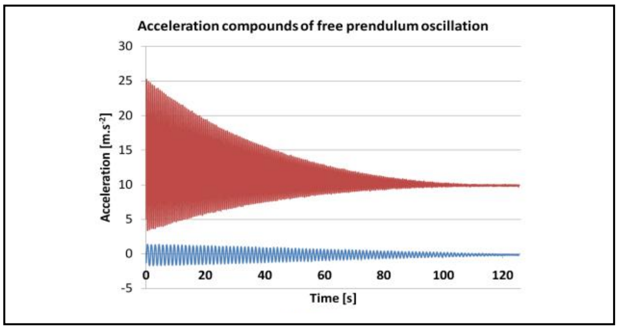

Fig. 3. Measured courses of pendulum acceleration compounds

The upper envelope of normal acceleration (*an*) is influenced by *g* value additive. We can see that *g* (9.81m.s^{-2}) is simultaneously the value of stabilization. Next with the use of equation (15) the amplitude of angle velocity can be expressed. The lower envelope presents extreme positions of pendulum ($\pm\varphi_{max}$), which is alternated in every T/2 time. Amplitudes of angle displacement is also possible to express from equation (16), when $\omega = 0$.

As we can see from the figure, the function of a_t (*at*) has a half frequency considering to a_n function. It lies on the fact that in one oscillation time (T) two a_n amplitudes of swings exist. Because of self sensor noise the a_t value carries information mainly about the direction (polarity) of actual displacement and therefore all next calculations is realized from a_n value.

4.2 Actual angle displacement identification

The actual angle displacement identification for purposes of industrial control, graphical representation or operations diagnostic can be in time of pendulum oscillation divided on three sections:

1. Identification up to time $2T$,
2. Identification from time $2T$ up to φ_m=5°,
3. Identification, when φ_m<5° up to 0°,

In the first time interval (up to 2T) the exact oscillating course isn't possible to predict. In second time interval (>$2T$) we can from two earlier displacement amplitudes express the damping coefficient value and period of oscillation. These parameters will be used for future prediction in next time period ($3T$). Simultaneously in $3T$ time we get the right value of angle displacement, which can be used for maximal error prediction computation. The difference between predicted and right value of displacement amplitude is this error by (15) equation. The error is computed only in each xT time (x (>3) is amplitude number).

$$\Delta\varphi_{(xT)} = \left(\varphi_p - \varphi_r\right)_{(xT)} \qquad (15)$$

By reason of sensor placing error elimination is the period and damping coefficient experimentally evaluated and compared for the displacement amplitude with the same polarity ($\pm\varphi_{max}$), hence in 2xT/2 and (2x-1)T/2 time. The figure (Fig. 4) shows the envelopes of right and predicted angle displacement as amplitudes extreme, which have been evaluated separately considering the polarity displacement (right and left).

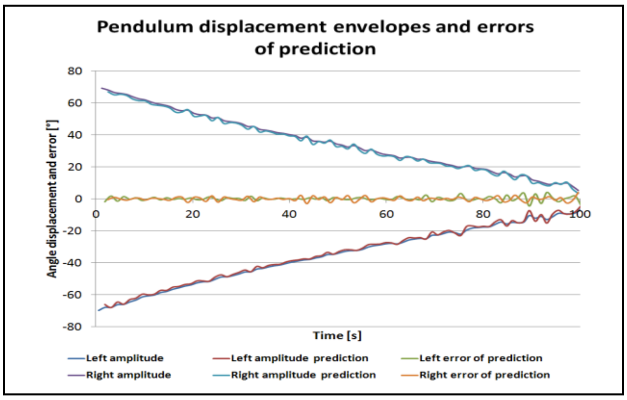

Fig. 4. Oscillating amplitude envelops – measured and predicted and prediction errors

From the functioncomparison is known, that the envelopes of predicted amplitude oscillation relatively well copies the right pendulum displacement. Also we have not confirmed the significant difference between right and left (odd and even amplitude) angle evaluation. This fact allows update of predicted parameters just in T/2 time using combined calculation. This approach could bring next reduction of prediction error.

The angle computation in the third time interval, thus from φ_m=5° to 0° is in large extend affected by the own sensor noise and small force actions of oscillating system. Therefore in this time interval the amplitude evaluation is incorrect. In this amplitude range we can use the simplification of computation by $\varphi = -a_n \cdot g^{-1}$ equation. But exact amplitude evaluation of this interval is needed to be considered.

5. EVALUATION

From the measurement that has been performed the average values are known. The main period of oscillation is T = 1.21s, damping coefficient is in average δ = -0.02577$rad.s^{-1}$ and error of displacement prediction is in average 0.000492rad. Total time of oscillating is 130.7s.

On the whole measurement process influences the large number of error parameters. One of these is the success of wireless transmission. By influence of error samples elimination the base sample frequency fluctuates from 98Hz to 104Hz. Wrong definition of the Kalman's filter parameters can causes the phase shift of sensed signals and also error in their proportional evaluation. The mechanical aspects of inaccuracy are asensor fixing error (axis alignment), friction in rotating storage and possible frame vibration.

We can suppose several limitations of the MEMS sensing methods and evaluation which is based on sampling frequency value, own noise of sensors and accumulation of integration error. Accuracy limitations of this approach are based on angle evaluation from goniometric function (arccos). Presented approach is useful only for harmonic oscillations with stationary frequency. For random oscillation (stochasticsignals) the method provides wrong results.

6. CONCLUSION

From the theoretical analysis and measurement which has been performed, we can say that this designed method is useful for purposes of graphical processes representation or oscillation parameter analysis in the field of control and regulation. As we identified, measurement accuracy of this approach varies in a wide range. For the better identification we should consider a more accurate method with using MEMS inclinometer or gyroscope. This way it would be possible to evaluate actual angle displacement in any time without prediction necessity.

7. REFERENCES

[1] Dado, S.& Kreidl, M. (1996). *Sensors and measuring circuits*, CVUT Praga, ISBN 80-01-01500-9, Praga

[2] Kurilla, J. (2010). Using of MEMS accelerometer for weight oscillation parameter identification of gantry crane. *Posterus*, Vol. 3, No. 8, (August 2010), pp. 1-12, ISSN 1338-0087

[3] SUNGWON, Y. & GERLA, M. (2010). *Energy efficient accelerometer data transfer for human body movement studies*, University of California / Department of Computer Science, Grand No. CNS-0917408

[4] Gajtanska, M.; Danihelova, A. & Nemec, M. (2005). *Physics*, TU in Zvolen, ISBN 80-228-1492-X, Zvolen

[5] Bodnar, F. (2001). *Mechanics I. Statics and kinematics*, TU in Zvolen, ISBN 80-228-0986-1, Zvolen

[6] Sotak, M.; Kralik, V. & Kmec, F. (2008). Inertial navigation for integrated navigational systems.*AT&P Journal*. Vol 18, No. 6, (June 2008), pp. 72-74, ISSN 1336-5010