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CLOSED LOOP STABILITY FOR DELAYED SYSTEMS: A CASE STUDY

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Abstract: *Internal delay is a natural part of many real-life processes and systems which are then called anisochronic. Transfer function representation of such systems yields a ratio of quasipolynomials, instead of polynomials, with delay (exponential) terms in denominators. Delay deteriorates the quality of feedback control performance. This contribution is focused on a rather detailed stability analysis of a closed loop characteristic quasipolynomial when stabilizing a selected anisochronic system.*
Key words: *Stability, delay systems, feedback control*

1. INTRODUCTION

Linear time delay systems and processes are usually considered with input-output delays only, which results in shifted arguments on the right-hand side of a differential equation. Modern control theory has been dealing with problem of delays since its nascence – the Smith predictor has been known for longer than five decades (Smith, 1957). Anisochronic systems and models (Bellmann & Cooke, 1963; Hale & Lunel, 1993; Zítek & Víteček, 1999), on the other hand, offer a more universal dynamics description applying delay elements on the left-hand side of a differential equation, which reflects a natural feature of many processes that they own delays in internal feedback loops (i.e. in state variables).

The Laplace transform results in the system transfer function as a ratio of so called quasipolynomials (El'sgol'ts & Norkin, 1973). This model has some interesting features, e.g. the spectrum has an infinite number of poles. The characteristic quasipolynomial appears also when dealing with the feedback control and thus it decides about the control system stability.

The aim of the presented paper is to investigate stability properties of a characteristic closed-loop quasipolynomial when control a selected anisochronic system by a simple proportional controller. Stability analysis is based on the important finding that the argument principle (i.e. the Mikhaylov criterion) holds for a class of retarded quasipolynomials represented by the studied one as well (Zítek, 1986; Górecki et al., 1989). The main result consists in the limitation for the controller parameter such that the closed loop remains stable. Notice that the investigated quasipolynomial was analyzed already in e.g. (Cooke & Grossman, 1982; Beretta & Kuang, 2002; Michiels et al., 2002); however, these authors utilized different approaches.

The presented principle can only be used when analyzing other retarded quasipolynomials or, using a modified Mikhaylov criterion, also for quasipolynomials of the neutral type.

2. CONTROL PROBLEM FORMULATION

Consider an anisochronic system described by the transfer function

$$G = \frac{b}{s + a\exp(-\tau s)} \tag{1}$$

and a proportional controller $R = r$, where $a,\, r \neq 0 \in \nabla$, $b,\, \tau > 0 \in \nabla$. Assume a simple feedback control scheme pictured in Fig. 1, where w is the reference signal, e represents the transformed control error, u expresses the plant input, and y stands for the plant output (controlled value).

The aim is to determine bounds for r so that the characteristic quasipolynomial

$$m(s) = s + br + a\exp(-\tau s) \tag{2}$$

of the closed loop is stable. As mentioned above, the standard Mikhaylov criterion can be utilized, the following relation holds for stable retarded quasipolynomials

$$\underset{s=\omega j,\,\omega\in[0,\infty]}{\Delta \arg} m(s) = \frac{n\pi}{2} \tag{3}$$

where n is the highest s-power in the quasipolynomial. Obviously, quasipolynomial (2) is stable iff the overall phase shift of the Mikhaylov plot is $\pi/2$.

3. STABILITY ANALYSIS

Due to the limited space only statements about stability of (2) based on (3) without proofs are presented below.

Lemma 1: For $\omega = 0$, the imaginary part of the Mikhaylov curve of quasipolynomial (2) equals zero and it approaches infinity for $\omega \to \infty$.

Lemma 2: If (2) is stable, the following inequality holds

$$r > \frac{-a}{b} \tag{4}$$

and thus the Mikhaylov curve begins on the positive real axis.

Lemma 3: A point on the Mikhaylov curve of (2) lies in the first quadrant for an infinitesimally small $\omega = \Delta > 0$ iff

$$a\tau \leq 1 \tag{5}$$

This point lies in the fourth quadrant iff

$$a\tau > 1 \tag{6}$$

Lemma 4: If the lower bound (4) holds and a, b, r are bounded, then $\mathrm{Re}\{m(\mathrm{j}\omega)\}$ is bounded for all $\omega > 0$.

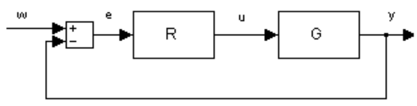

Fig. 1. Feedback control system

Lemma 5: There it exists an intersection of the Mikhaylov plot with the imaginary axis for some $\omega > 0$ iff $a \geq 0$ and

$$|br| \leq |a| \tag{7}$$

Searching of the stability upper bound will be made in two branches, so that conditions (5) and (6) are solved separately. The following theorem presents the necessary and sufficient stability condition for the former case.

Theorem 1: If (5) holds, then quasipolynomial (2) is stable iff condition (4) is satisfied.

Now consider the second case, i.e. $a\tau > 1$. The following result reinforces condition (4). Define the crossover frequency ω_C as the least positive frequency where the curve crosses the real axis, i.e.

$$\operatorname{Im}\{m(\mathrm{j}\omega_C)\} = 0 \Leftrightarrow a = \frac{\omega_C}{\sin(\tau\omega_C)} \tag{8}$$

Theorem 2: If (6) holds, then quasipolynomial (4) is stable iff

$$r > \frac{-a\cos(\tau\omega_C)}{b} \tag{9}$$

where $\omega_C > 0$ is given by (8).

4. EXAMPLES

The following examples demonstrate and verify above statements.

First, let the unstable plant transfer function be of the form

$$G = \frac{1}{s - \exp(-0.8s)} \tag{10}$$

i.e. $a\tau \leq 1$, hence the admissible proportional controller according to (4) reads $r > 1$. Let $r = 2$, then the corresponding Mikhaylov plot of the characteristic quasipolynomial and the stabilized closed loop response, respectively, are displayed in Fig 2.

Second, consider the unstable plant giving rise to the transfer function

$$G = \frac{1}{s + 2\exp(-0.8s)} \tag{11}$$

which satisfies (6). The critical frequency is $\omega_C \doteq 2$ [rad·s^{-1}] and thus the closed loop is stable iff $r > 5.71 \cdot 10^{-2}$. Choose e.g. $r = 1$, then the corresponding Mikhaylov plot of the characteristic quasipolynomial and the stabilized closed loop response, respectively, are displayed in Fig 3.

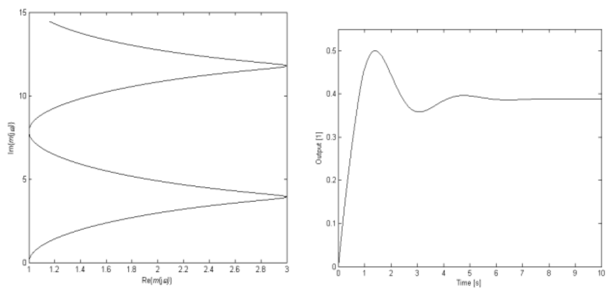

Fig. 2. The Mikhaylov plot of $m(s)$ and the corresponding closed loop response when control (10) by the proportional controller $r = 2$

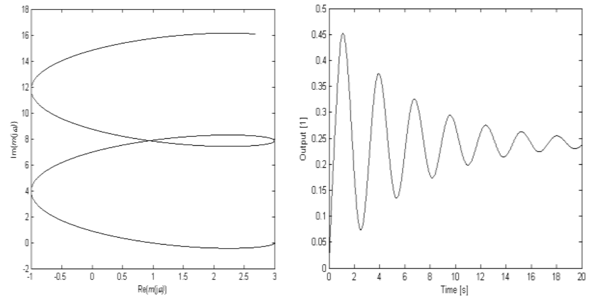

Fig. 3. The Mikhaylov plot of $m(s)$ and the corrsponding closed loop response when control (11) by the proportional controller $r = 1$

5. CONCLUSION

Stability analysis of a closed loop characteristic quasipolynomial when controlling an anisochronic system by a proportional controller has been presented in this paper. The aim has been to find acceptable limits for a proportional controller based on the argument principle. Unproven statements only have been presented because of the space limit. Simulation examples figure the Mikhaylov plot of a stable characteristic quasipolynomial together with control stabilized responses. These examples demonstrate that appropriate selected controllers stabilize the closed loop systems. The analytic tools utilized in this contribution can be employed when studying other retarded quasipolynomials as well.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

Bellman, R. & Cooke, K. L. (1963). *Differential-difference Equations*, Academic Press, New York

Beretta, E. & Kuang, Y. (2002). Geometric stability switch criteria in delay differential systems with delay depended parameters. *SIAM Journal on Mathematical Analysis*, Vol. 3, No. 5, 1144-1165

Cooke, K. L. & Grossman, Z. (1982). Discrete delay, distributed delay and stability switches. *Journal of Mathematical Analysis and Applications*, Vol. 86, 592-627

El'sgol'ts, L. E. & Norkin, S. B. (1973). *Introduction to the Theory and Application of Differential Equations with Deviated Arguments*, Academic Press, New York

Górecki, H.; Fuksa, S.; Grabowski, P. & Korytowski, A. (1989). *Analysis and Synthesis of Time Delay Systems*, John Wiley & Sons, ISBN 978-0471276227

Hale, J. K. & Verduyn Lunel, S. M. (1993). *Introduction to Functional Differential Equations*, Applied Math. Sciences 99, Springer-Verlag, ISBN 0-387-94076-6, New York

Michiels, W.; Engelborghs, K.; Vansevenant, P. & Roose, D. (2002). Continuous pole placement for delay equations. *Automatica*, Vol 38, No. 5, 747-761

Smith, O. J. M. (1957). Closer control of loops with death time. *Chem. Eng. Prog*., Vol. 53, No. 5, 1957, 217-219

Zitek, P. (1986). Anisochronic modelling and stability criterion of hereditary systems. *Problems of Control and Information Theory*, Vol. 15, No. 6, 413-423

Zitek, P. & Vitecek, A. (1999). *The Control Design of Subsystems with Delays and Nonlinearities* (in Czech), CVUT Publishing, ISBN 80-01-01939-X, Prague