# STRUCTURAL AND KINEMATIC MODELING OF A QUADRUPED BIOMECHANISM 

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#### Abstract

In this paper we present the kinematic model of a biomechanism which represent the legs of a four legged mammal. The anterior legs and posterior are realized as plane mechanisms, with articulated bars. Each anterior leg has a complex structure with three closed contours, mean while each posterior leg has only two closed contours. Each mechanism is actuated by an electric motor. The geometric and kinematic modeling of the anterior leg mechanism is achieved by means of some vectorial and scalar equations. Also, the kinematic simulation is achieved by means of ADAMS software, considering as basis, the upper platform of each mechanism. Key words: biomechanism, kinematic, modeling.


## 1. INTRODUCTION

In case of four legged mammals, the structure of anterior and posterior legs is very similarly by the structure of most majorities of actual four legs quadrupeds (Buzea, 2005). To some quadrupeds, the anterior legs are short that those posterior. To remark, that at quadrupeds, the anterior legs have the degree of mobility larger than the posterior legs.

By physical modeling of a dog (fig. 1) we obtain a biomechanism (mobile biorobot), in which the legs are realized like plane articulated kinematic chains (Antonescu\&Buzea, 2005).


Fig. 1. The picture of the dog as a quadruped biomechanism

## 2. KINEMATIC SCHEME AND THE MOBILITY OF THE BIOMECHANISM

The kinematic scheme of the quadruped biomechanism is realized in vertical longitudinal plane (fig. 2), in which are represented the plane articulated mechanisms of those two legs, from rear (fig. 2a) and front (fig. 2b). The booth mechanisms are articulated in the upper side to a horizontal link, which represent the body of the physically modeled dog.

The joints $A_{0}$ and $B_{0}$ of each mechanism to the upper mobile platform (fig. 2) are considered as basis joints, by this reason this platform has been noted with 0 .

Each from those two mechanisms (rear and front) has a first kinematic chain, the four bar mechanism $A_{0} \mathrm{ABB}_{0}$, which is formed from the kinematics chains $0,1,2$ and 4 . The second
kinematic chain of each mechanism is the four bar articulated mechanism ACED, with the kinematic elements 1, 2, 4 and 5 (fig. 2a) or BCED, from the elements 2, 3, 4 and 5 (fig. 2b). The mechanism of the front leg contain another kinematic chain DGHF (fig. 2b), which is formed from the kinematic elements $2,5,6$ and 7 .

The mobility $M_{b}$ of each from those two plane mechanisms is calculated with the Dobrovolski formula:

$$
\begin{equation*}
M_{b f}=(6-f) n-\sum_{k=f+1}^{5}(k-f) C_{k} \tag{1}
\end{equation*}
$$

which for $f=3$ (plane mechanisms) become the Grübler-Cebâşev formula:

$$
\begin{equation*}
M_{b 3}=3 n-\sum_{k=4}^{5}(k-3) C_{k}=3 n-2 C_{5}-C_{4} \tag{2}
\end{equation*}
$$

where the class of the kinematic joint distinguish the imposed restrictions ( $k=5, k=4$ ).

To calculate the mobility of those two mechanisms (fig. $2 \mathrm{a}, 2 \mathrm{~b}$ ) we use the formulas (1) and (2):

$$
\text { a) } M_{b 3}=3 n-2 C_{5}-C_{4}=3 \times 5-2 \times 7-0=1
$$

$$
M_{b}=C_{1}+2 C_{2}-3 N_{3}=7+2 \times 0-3 \times 2=1 .
$$

b) $M_{b 3}=3 n-2 C_{5}-C_{4}=3 \times 7-2 \times 10-0=1$;
$M_{b}=C_{1}+2 C_{2}-3 N_{3}=10+2 \times 0-3 \times 3=1$.


Fig. 2. Kinematical scheme of the mechanisms from posterior legs (a) and anterior (b)

## 3. KINEMATIC MODELLING OF THE ANTERIOR LEG

The mechanism has three independent contours (fig. 2b) 1) $\left.-\mathrm{A}_{0} \mathrm{~B}_{0} \mathrm{BAA}_{0}(03210), 2\right)-\mathrm{BCEDB}$ (34523) and 3) GDFHG (25762). We choose a coordinate system with the origin in the fixed joint $\mathrm{A}_{0}$ (fig. 3), having the axis $\mathrm{A}_{0} \mathrm{x}$ and $\mathrm{A}_{0} \mathrm{y}$ orientated from right to left, respectively from upper to bottom.

The closing vectorial equation of the first contour (03210) is writhed explicitly (fig. 3):

$$
\begin{equation*}
\overrightarrow{B_{0} A_{0}}+\overrightarrow{A_{0} A}=\overrightarrow{B_{0} B}+\overrightarrow{B A} \tag{3}
\end{equation*}
$$



Fig. 3. The vectorial configuration of the first kinematic chain
We arrange the terms of equations (3), that in the left part to be the vectors which contain the unknown (the angles $\varphi_{2}$ and $\varphi_{3}$ ), and in the right side to be the vectors known as size and direction (the angle $\varphi_{I}$ is the independent parameter, being imposed in certain give interval):

$$
\begin{equation*}
\overrightarrow{B A}+\overrightarrow{B_{0} B}=\overrightarrow{B_{0} A_{0}}+\overrightarrow{A_{0} A} \tag{3'}
\end{equation*}
$$

We introduce the notations: $\overrightarrow{B_{0} A_{0}}=\vec{l}_{0}, \overrightarrow{A_{0} A}=\vec{l}_{1}$, $\overrightarrow{B A}=\vec{l}_{2}, \overrightarrow{B_{0} B}=\vec{l}_{3}$, else the vectorial equation (3') is writhed under a convenient form:

$$
\begin{equation*}
\vec{l}_{2}+\vec{l}_{3}=\vec{l}_{0}+\vec{l}_{1} \tag{4}
\end{equation*}
$$

Projecting the vectorial contour on the coordinate's axis $A_{0} x$ and $A_{0} y$ (fig. 4) we obtain two scalar equations equivalent to the vectorial equation (5):

$$
\left\{\begin{align*}
l_{2} \cos \varphi_{2}+l_{3} \cos \varphi_{3} & =l_{0} \cos \varphi_{0}+l_{1} \cos \varphi_{1}  \tag{5}\\
l_{2} \sin \varphi_{2}+l_{3} \sin \varphi_{3} & =l_{0} \sin \varphi_{0}+l_{1} \sin \varphi_{1}
\end{align*}\right.
$$

The nonlinear system of equations (5) can be resolved by eliminating one of two unknown $\varphi_{2}$ and $\varphi_{3}$.

For that the system is writhed more compactly, under the form:

$$
\left\{\begin{array}{l}
l_{2} \cos \varphi_{2}+l_{3} \cos \varphi_{3}=b_{1}\left(\varphi_{1}\right) \\
l_{2} \sin \varphi_{2}+l_{3} \sin \varphi_{3}=b_{2}\left(\varphi_{1}\right)
\end{array}\right.
$$

where we have used the notations:
$b_{1}\left(\varphi_{1}\right)=l_{0} \cos \varphi_{0}+l_{1} \cos \varphi_{1} ;$
$b_{2}\left(\varphi_{1}\right)=l_{0} \sin \varphi_{0}+l_{1} \sin \varphi_{1}$.
To calculate the angle $\varphi_{2}$ we isolate the terms which contain the other unknown $\varphi_{3}$ :

$$
\left\{\begin{array}{l}
l_{3} \cos \varphi_{3}=b_{1}-l_{2} \cos \varphi_{2}  \tag{6}\\
l_{3} \sin \varphi_{3}=b_{2}-l_{2} \sin \varphi_{2}
\end{array}\right.
$$

After square up of those two equations (6) and summing, it results:

$$
\begin{equation*}
l_{3}^{2}=l_{2}^{2}+b_{1}^{2}+b_{2}^{2}-2 l_{2} b_{1} \cos \varphi_{2}-2 l_{2} b_{2} \sin \varphi_{2} \tag{7}
\end{equation*}
$$

The obtained expression (8) is a trigonometrically equations with variable coefficients, under the form:

$$
\begin{equation*}
A_{1}\left(\varphi_{1}\right) \sin \varphi_{2}+B_{1}\left(\varphi_{1}\right) \cos \varphi_{2}+C_{1}\left(\varphi_{1}\right)=0 \tag{8}
\end{equation*}
$$

where the variable coefficients have the expressions:

$$
\begin{align*}
& A_{1}\left(\varphi_{1}\right)=2 l_{2} b_{2}\left(\varphi_{1}\right) ; B_{1}\left(\varphi_{1}\right)=2 l_{2} b_{1}\left(\varphi_{1}\right) ;  \tag{9}\\
& C_{1}\left(\varphi_{1}\right)=l_{3}^{2}-l_{2}^{2}-b_{1}^{2}\left(\varphi_{1}\right)-b_{2}^{2}\left(\varphi_{1}\right)
\end{align*}
$$

With the help of formulas $\sin \varphi=\frac{2 \operatorname{tg} \frac{1}{2} \varphi}{1+t^{2} \frac{1}{2} \varphi} \cos \varphi=\frac{1-\operatorname{tg}^{2} \frac{1}{2} \varphi}{1+t_{g}^{2} \frac{1}{2} \varphi}$ the solutions of the equations (7) are deducted under the form:

$$
\begin{equation*}
\varphi_{2}=2 \operatorname{arctg}\left(\frac{A_{1} \pm \sqrt{A_{1}^{2}+B_{1}^{2}-C_{1}^{2}}}{B_{1}-C_{1}}\right) \tag{10}
\end{equation*}
$$

## 4. DIAGRAMS OF ANGULAR DISPLACEMENTS VARIATIONS

We consider the uniform movement of the motor element 1 (fig. 3). With the help of the MSC.ADAMS software we simulate the movement of the anterior leg for the angle $\varphi_{1}=$ $52^{\circ}$.


Fig. 4 . Law of variation for the angle $\varphi_{2}$


Fig. 5. Law of variation for the angle $\varphi_{3}$


Fig. 6. Trajectory of $F$ joint between elements 6 and 7

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