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A GRAPHICAL APPROACH TO ROBUST STABILITY ANALYSIS OF DISCRETE-TIME SYSTEMS WITH PARAMETRIC UNCERTAINTY

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Abstract: *The contribution deals with a graphical approach to investigation of robust stability of discrete-time polynomials with parametric uncertainty. The applied techniques include the value set concept and general version of the zero exclusion condition for robust D-stability. The analyses are accomplished for both robustly stable and robustly unstable cases using the Polynomial Toolbox for Matlab.*
Key words: *Robust Stability Analysis, Discrete-Time Systems, Value Set Concept, Polynomial Toolbox*

1. INTRODUCTION

Many practically used control design methods start from a mathematical model of controlled process. This model is usually constructed in order to be as simple as possible, i.e. low order linear time invariant one, although the real process is much more complex. Frequently, "less important" properties especially from the realms of fast dynamics, nonlinearities or time-variant behaviour are neglected during modelling and they have to be somehow taken into consideration later on. This is the space for introducing the uncertainty into the model description. Nonetheless, the presence of uncertainty should not be excluded even if the processes are basically linear, because the physical parameters are never known exactly, or they can vary in accordance with operating conditions. A convenient way of including the uncertainty into the model consists in use of parametric uncertainty (Kučera, 2001; Matušů & Prokop, 2008; Matušů &Prokop, 2010).

The cardinal task of all controller design techniques is to assure the stability of closed control loop. Besides, if the stability is required not only for one nominal system but for the whole family of systems defined by its uncertain model, one speaks about robust stability. Analysis of robust stability represents deeply studied discipline in the last decades. Thus, lots of possible investigation methods have appeared (Ackerman, *et al.*, 1993; Barmish, 1994; Bhattacharyya, *et al.*, 1995; Sánchez-Peña & Sznaier, 1998). The elegant, effective and generally applicable graphical approach employs the value set concept in combination with zero exclusion condition.

The main aim of this paper is to present a possible graphical approach to robust *D*-stability analysis based on the value set concept and general version of the zero exclusion condition with emphasis on its application to discrete-time systems. The examples are plotted with the assistance of the Polynomial Toolbox for Matlab (www.polyx.com; Šebek, *et al.*, 2000).

2. PROBLEM FORMULATION

From a control viewpoint, a common object of investigation is the uncertain characteristic polynomial of the discrete-time closed-loop control system. It can be generally described by:

$$p(z,q)=\sum_{i=0}^{n}\rho_i(q)z^i \qquad (1)$$

where z is the complex variable, q is a vector of uncertain parameters and ρ_i are coefficient functions. The vector q is supposed to be confined by an uncertainty bounding set Q, which is usually given a priory, e.g. by operating conditions, and which will be considered as a box in this paper.

The task is to analyse the robust stability of polynomial (1). It is well known that the fixed discrete-time polynomial $p(z)$ is stable if and only if all its roots are inside the unit circle with the centre in the origin of the complex plane. The family of discrete-time polynomials:

$$P=\{p(z,q):q\in Q\} \qquad (2)$$

is robustly stable if and only if $p(z,q)$ is stable for all $q\in Q$.

The way how the uncertain parameters enter into the coefficients of the polynomial via coefficient functions $\rho_i(q)$ is very significant because it influence the choice of tool for robust stability analysis. Generally, the selection is better for the continuous-time cases, mainly thanks to number of results based on fundamental Kharitonov theorem. Unfortunately, the Kharitonov-like extremal results are not generally available for discrete-time systems. Besides the existence of several analytical methods, the very universal graphical approach consisting of the value set concept and the zero exclusion condition can be advantageously utilized here.

3. VALUE SET CONCEPT AND ZERO EXCLUSION CONDITION

The specialized continuous-time version of the value set concept and the zero exclusion condition can be found e.g. in (Barmish, 1994; Matušů & Prokop, 2009). However, the concept has been extended and generalized to so-called robust *D*-stability framework (Barmish, 1994) which allows to investigate robust stability for an arbitrary stability region *D*.

Assume a family of polynomials $P=\{p(\cdot,q):q\in Q\}$. The value set at an arbitrary evaluation point $x\in\mathbf{C}$ is given by:

$$p(x,Q)=\{p(x,q):q\in Q\} \qquad (3)$$

In other words, $p(x,Q)$ is the image of Q under $p(x,\cdot)$. For example, in discrete-time case substitute z for a point at the unit circle in a family $P=\{p(z,q):q\in Q\}$ and let the vector of uncertain parameters q range over the set Q.

The zero exclusion condition formulated in (Barmish, 1994)) says: Let D be an open subset of the complex plane and assume that $P=\{p(\cdot,q):q\in Q\}$ is a family of polynomials with invariant degree, uncertainty bounding set Q which is pathwise connected. Moreover, suppose that the coefficient functions $\rho_i(q)$ are continuous and that P has at least one D-stable member $p(\cdot,q_0)$. Then P is robustly D-stable if and only if

$$0 \notin p(x,Q) \tag{4}$$

for all $x \in \partial D$, where ∂D denotes the boundary of D.

4. ILLUSTRATIVE EXAMPLE

Consider the fifth order discrete-time interval polynomial:

$$p(z,q) = [1,2] + [3,4]z + [5,6]z^2 + [7,8]z^3 + [9,10]z^4 + [11,12]z^5 \tag{5}$$

and decide on its robust stability.

For the purpose of testing in the Polynomial Toolbox (www.polyx.com; Šebek, *et al.*, 2000), the polynomial (5) has been rewritten to the polytopic form:

$$p(z,q) = p_0(z) + q_1 p_1(z) + q_2 p_2(z) + q_3 p_3(z) + q_4 p_4(z) + q_5 p_5(z) \tag{6}$$

where:

$$\begin{aligned} &p_0(z) = 1 + 3z + 5z^2 + 7z^3 + 9z^4 + 11z^5 \\ &p_1(z) = 1; \quad p_2(z) = z; \quad p_3(z) = z^2 \\ &p_4(z) = z^3; \quad p_5(z) = z^4; \quad p_6(z) = z^5 \end{aligned} \tag{7}$$

and $q_i \in \langle 0,1 \rangle$.

So, the simple sequence of commands was typed in Matlab with Polynomial Toolbox:

```
pinit
p0 = 1 + 3*z + 5*z^2 + 7*z^3 + 9*z^4 + 11*z^5;
p1 = 1; p2 = z; p3 = z^2; p4 = z^3; p5 = z^4; p6 = z^5;
Qbounds = [0 1; 0 1; 0 1; 0 1; 0 1; 0 1];
isstable(p0)
ptopplot(p0, p1, p2, p3, p4, p5, p6, Qbounds,…
   exp(j*(0:0.001:1)*2*pi))
```

The routine "isstable" verifies the stability of one family member $p_0(z)$ and command "ptopplot" draws the value sets as shown in fig. 1. As can be seen, the origin of the complex plane is excluded from the value sets and consequently the polynomial (5) is robustly stable.

Now consider e.g. an extension of bound in interval parameter by z^4 from polynomial (5):

$$p(z,q) = [1,2] + [3,4]z + [5,6]z^2 + [7,8]z^3 + [9,19]z^4 + [11,12]z^5 \tag{8}$$

Analogical process leads to fig. 2 with relevant value sets. In this case, the zero point is included in the value sets which means that the polynomial (8) is not robustly stable.

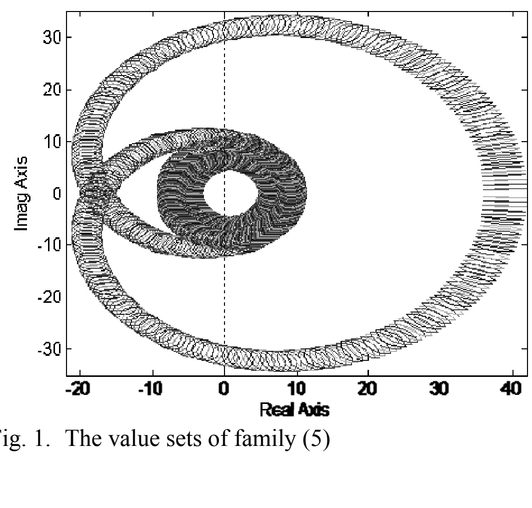

Fig. 1. The value sets of family (5)

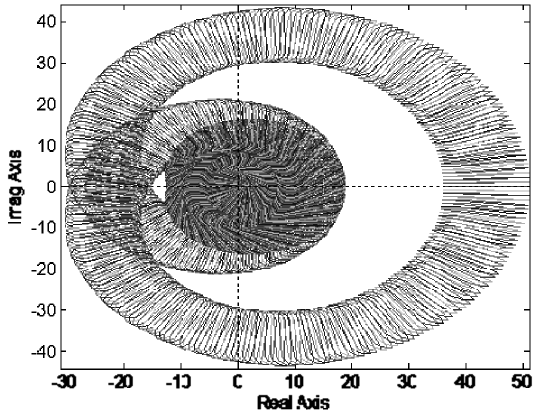

Fig. 2. The value sets of family (8)

5. CONCLUSION

The main aim of the contribution has been to present a possible graphical approach to analysis of robust stability for discrete-time systems with parametric uncertainty based on the value set concept and general version of the zero exclusion condition. Two examples for both robustly stable and unstable cases were analysed using the Polynomial Toolbox for Matlab.

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