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ROBUST METHODS FOR DIAGNOSIS IN SYSTEMS WITH NONLINEAR LINKS

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Abstract: *The problem of fault diagnosis in dynamic systems described by structural schemes including blocks with transfer functions and static nonlinearities is studied. The fault isolation observer-based procedure with an accuracy of a block is developed. The approach for diagnosis in nonlinear systems based on special canonical form similar to the Kronecker canonical form for linear systems is presented.*
Key words: *Systems, nonlinearities, diagnosis, observer, robustness*

1. INTRODUCTION

The problem of robust fault detection and isolation was extensively investigated for the past 20 years; see, e.g., the papers (Low et al., 1986; Frank, 1990), the books (Patton et al., 2000; Blanke et al., 2003). Wide class of different systems is considered: linear, linear analytical, polynomial, nonlinear, descriptor.

In this paper we consider a class of nonlinear systems described by structural schemes including, in addition to blocks with transfer functions, blocks with static nonlinearities describing such types of nonlinearities as backlash, saturation, hysteresis, and Coulomb friction. By analogy with linear systems, the description of this class is as follows:

$$z(p) = F(p)z(\mathrm{p}) + G(p)u(p) + K\rho(p) \tag{1}$$

where the vector $z \in Z \subseteq R^n$ will be named the state vector of this model, $u \in U \subseteq R^m$ is the vector of control; elements of matrices F and G are transfer functions (TF) and static nonlinearities, p is the differentiation operator. The term $K\rho(p)$ models the unknown inputs to the actuator and to the dynamic process, the evaluation of the q-dimensional vector function ρ is considered unknown. The term $K\rho(p)$ will be named the factor ρ.

It is supposed that products in (1) should be interpreted as follows: if F_{ij} is TF, then $F_{ij}(p)z_j(p)$ is a product of $F_{ij}(p)$ by the variable $z_j(p)$; if F_{ij} is a static nonlinearity, then $F_{ij}z_j(p)$ is the nonlinear function F_{ij} with the argument $z_j(p)$. Here F_{ij} is the j-th element of the i-th row of the matrix F, $z_j(p)$ is the j-th component of the vector variable $z(p)$. Therefore every term F_{ij} describes some block (TF or nonlinearity), and every block is some single input – single output subsystem which presents the linear dynamic (or nonlinear static) one.

Without loss of generality assume that $y(p) = Hz(p)$ where each row of the matrix H is of the form [0…1…0] with single 1 in a certain position.

The problem is to design the robust diagnostic procedure isolating faults with an accuracy of a block.

2. ANALYSIS OF FACTOR ρ

To solve this problem, recall that in (Yakshin & Zhirabok, 2005) the following description of the observer was used:

$$z^*(p) = F^*(p)z^*(\mathrm{p}) + G^*(p)u(p) + S(p)y(p)\,,$$
$$y^*(p) = H^*z^*(p)\,. \tag{2}$$

This observer generates the residual

$$r(p) = Ry(p) - y^*(p)$$

for some matrix R. It is supposed that

$$Tz(p) = z^*(p) \tag{3}$$

for some matrix T. Consider the affect of the factor ρ on the difference $e(p) = Tz(p) - z^*(p)$ which can be analyzed via replacing the variables $z(p)$ and $z^*(p)$ with right-hand sides of equations (1) and (2) using the equality $Tz(p) = z^*(p)$:

$$\begin{aligned} &e(p) = TFz(p) + TGu(p) + TK\rho(p) - \\ &F^*z^*(p) - G^*u(p) - SHz(p) = \\ &F^*Tz(p) - F^*z^*(p) + TK\rho(p). \end{aligned}$$

Clearly, the term $TK\rho(p)$ describes the influence of the factor ρ on the difference $e(p)$. It can be shown that $r = H^*e$ in the linear and nonlinear cases. Therefore, to minimize the influence of ρ on r, the influence of ρ on e must be minimized. The ideal case $TK = 0$ is known as full decoupling. In practice, this case is frequently impossible therefore partial decoupling has to be used.

To achieve partial decoupling, the approach based on singular value decomposition of K given by

$$K = U_k \Sigma V_k$$

suggested in (Low et al., 1986) will be used. Here U_k and V_k are orthogonal matrices, $\Sigma_L = [diag(\sigma_1, \sigma_2, ..., \sigma_n)\ 0]$, $0 \le \sigma_1 \le \sigma_2 \le ... \le \sigma_n$ are singular values of the matrix K. The optimal choice of T is when the first s columns of the matrix U_k are used as rows of the matrix T. In this case the Frobenius norm of the product TK, i.e. $\|TK\|_F = (\mathbf{tr}(TK(TK)^T))^{1/2}$ is equal to

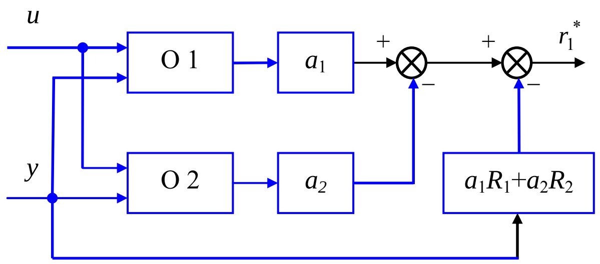

Fig. 1. Joint work of two observers for residual generation

$$\varepsilon_\rho = \|TK\|_F = (\sigma_1^2 + \sigma_2^2 + ... + \sigma_s^2)^{1/2} . \qquad (4)$$

Clearly, if $TK = 0$, then $\|TK\|_F = 0$.

3. SUGGESTED APPROACH DESCRIPTION

In the approach suggested in (Yakshin & Zhirabok, 2005) to observer design, the matrix T is uniquely determined, and it is independent of the matrix K. To solve the problem of partial decoupling in this case, a new approach is suggested. It will be illustrated by the example when three observers have been built for diagnosis in a given system.

The main idea of this approach is illustrated in Fig. 1 where two observers (O 1 and O 2) are used to generate the residual r_1^*. Here R_1 and R_2 are row matrices, a_1 and a_2 are some coefficients which must be found based on the following idea. The influence of the factor ρ on the residual r_1^* can be specified by analogy with (4):

$$\varepsilon_\rho^2 = \left\| a_1 T^{(1)} K + a_2 T^{(2)} K \right\|_F^2$$

where $T^{(1)}$ and $T^{(2)}$ are matrices describing the first and second observers by analogy with (3). It can be shown that

$$\varepsilon_\rho^2 = a_1^2 s_1 + 2a_1 a_2 s_{12} + a_2^2 s_2$$

where

$$s_1 = \left\| T^{(1)} K \right\|_F^2 , \qquad s_2 = \left\| T^{(2)} K \right\|_F^2 ,$$

$$s_{12} = \sum_{i=1}^{k} \sum_{j=1}^{q} (\sum_{v=1}^{n} T_{iv}^{(1)} K_{vj})(\sum_{v=1}^{n} T_{iv}^{(2)} K_{vj}) .$$

If these observers have different dimensions, $T^{(1)}K$ and $T^{(2)}K$ have different numbers of rows. Therefore the matrix $T^{(1)}$ or $T^{(2)}$ must be supplemented with zero rows. Denote the number of rows in these matrices by k.

Minimization of ε_ρ^2 gives $a_1 = a_2 = 0$ as a rule (except the case $T^{(1)}K = aT^{(2)}K$ for some a that gives $a_1 = 1$, $a_2 = -a$). To solve this problem, consider the influence of the faults ε_f on the residual r_1^* that can be specified in the simplest case as $\varepsilon_f^2 = a_1^2 + a_2^2$ and maximize this influence. The influence of the factor ρ (i.e. the value ε_ρ) in this case must be limited by certain number σ. By this means, the constrained optimization problem can be formulated as follows:

$$a_1^2 + a_2^2 \to \max ,$$
$$a_1^2 s_1 + 2a_1 a_2 s_{12} + a_2^2 s_2 \le \sigma .$$

To solve this problem, Lagrange multiplier technique is used that gives an unconstrained optimization problem:

$$\Phi(a_1, a_2) = a_1^2 + a_2^2 + \lambda(a_1^2 s_1 + 2a_1 a_2 s_{12} + a_2^2 s_2 - \sigma) \to \max ,$$

λ is a Lagrange multiplier parameter.

It can be shown that this task results in the following equations:

$$s_{12}(a_1^2 - a_2^2) + a_1 a_2 (s_2 - s_1) = 0 ,$$
$$a_1^2 s_1 + 2a_1 a_2 s_{12} + a_2^2 s_2 = \sigma .$$

An analytical solution of these equations is unwieldy; the numerical one can be obtained on the basis of mathematical packages.

By this means, the pair {first, second} observers generates the residual r_1^*; by analogy, one can use the pairs {second, third} and {first, third} observers to generate the residuals r_2^* and r_3^*, respectively. It can be shown that quality of diagnosis is not impaired in this case.

4. CONCLUSIONS

The paper deals with the problem of the observer-based fault detection and isolation in dynamic systems described by structural schemes including blocks with transfer functions and static nonlinearities. Using a bank of observers, fault isolation will be performed with an accuracy of a set of blocks corresponding to some observer. In some cases this accuracy can be improved. The approach to minimize the influence of the unknown inputs factor based on using pairs of observers has been suggested.

Next steps of this research are diagnosis in dynamic systems with transfer functions containing nonlinear blocks described by the product $z_i(p) z_j(p)$ and blocks with multy inputs.

5. ACKNOWLEDGMENTS

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