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FAULT ACCOMMODATION IN BILINEAR DYNAMIC SYSTEMS

FILARETOV, V[ladimir]; ZHIRABOK, A[lexey]; SHUMSKY, A[lexey] & BOBKO, Y[evgeny]

Abstract: *Solution to the problem of fault accommodation in bilinear dynamic systems is related to constructing the control law which provides full decoupling with respect to fault effects. Existing conditions are formulated and calculating relations are given for the control law.*
Key words: *Systems, nonlinearities, accommodation, faults, diagnosis*

1. INTRODUCTION

An increasing demand on reliability and safety for critical purpose control systems calls for the use of fault tolerant control (FTC) techniques. The goal of FTC is to determine such control law which preserves the main performances of the system when a fault occurs. There are two principle approaches to FTC (Blanke et al., 2003; Noura et al., 2009). The first one is self-tuning or fault accommodation. It is related to on-line control law determination that preserves the main performances of the system in faulty case while the minor performances may degrade. The second way is self-organization which involves the system reconfiguration to replace the faulty parts of the system with the healthy ones. In (Shumsky & Zhirabok, 2009) a solution to the accommodation problem in nonlinear systems has been obtained on the basis of algebra of functions and differential geometry. In present paper, this problem is solved for bilinear systems. These systems form an important class of nonlinear systems. They are used to represent a wide variety of processes and systems including nuclear reactors, fermentation processes, hydraulic systems, *etc* (Shields, 1995).

Consider nonlinear system Σ described by the equations

$$\dot{x}(t) = Fx(t) + Gu(t) + \sum_{i=1}^{m} u_i(t)F^i x(t) + L\vartheta(t), \quad y(t) = Hx(t). \quad (1)$$

Here *x*, *y*, and *u* are the vectors of state, output, and control, respectively; F, G, H, L, F^1, …, F^m are known matrices of appropriate dimensions; $\vartheta(t) \in R^v$ is the vector describing the faults. Assume that for the healthy system the equality $\vartheta(t) = 0$ holds. The system (1) without the nonlinear term will be named the linear part of (1).

It is assumed that fault detection procedure is performed by known methods. If a fault occurs, $\vartheta(t)$ becomes an unknown function, and a solution to the control problem based on model (1) becomes impossible. To overcome this difficulty, it is suggested to obtain the vector $u(t)$ according to the relation

$$u(t) = g(y(t), x_0(t), u_*(t)) \quad (2)$$

for some function *g* where $u_*(t) \in R^m$ is a new control vector, $x_0(t) \in R^s$, $s \le n$, is a state vector of the system has to be determined and described by the equation

$$\dot{x}_0(t) = F_0 x_0(t) + G_0 u(t) + J_0 y(t) + \sum_{i=1}^{m} u_i(t) F_0^i x_0(t). \quad (3)$$

Model (3) does not depend on unknown vector $\vartheta(t)$ and can be used to design the observer for estimating the system state vector when the fault occurred. Assume that the model obtained by substation (2) into (1) can be transformed to the form

$$\dot{x}_*(t) = F_* x_*(t) + G_* u_*(t) + \sum_{i=1}^{m} u_i(t) F_*^i x_*(t)) \quad (4)$$

with $x_*(t) \in R^p$, $p \le q$. If the control (2) exists and the fault occurred and detected, then a solution to the control problem is performed on the basis of model (4) which does not contain the unknown vector $\vartheta(t)$. As a result, fault accommodation effect is achieved. Scheme for the system Σ control is shown in Fig. 1. The problem is to determine the existing condition for control (2) and to obtain matrices describing the systems (3) and (4).

The limitation of the suggested approach is that the problem of fault accommodation can not be solved for some faults. In these cases one has to use self-organization.

2. PRELIMINARY RESUTS

So-called logic-dynamic approach (Zhirabok & Usoltsev, 2002) will be used for solving fault accommodation problem. The feature of this approach is the use of conventional linear algebraic tools instead of nonlinear algebraic and differential geometric tools used in (Shumsky & Zhirabok, 2009). In the first step of the logic-dynamic approach, replace the system (1) with a system with transformed bilinear term as follows:

$$\dot{x}(t) = Fx(t) + Gu(t) + \sum_{i=1}^{m} u_i(t) \sum_{j=1}^{n} G^j F^{ij} x(t) + L\vartheta(t), \quad (5)$$

where F^{ij} is the *j*-th row of the matrix F^i, $G^1 = (1\ 0\ \ldots\ 0)$, $G^2 = (0\ 1\ \ldots\ 0)$, …, $G^n = (0\ 0\ \ldots\ 1)$.

In the second step, a linear part of the system (3) is designed. It is well-known from the theory of linear systems diagnosis that the matrix Φ exists such that $\Phi x(t) = x_0(t)$ in the unfaulty case. In the absence of faults, the following set of equations holds (Zhirabok & Usoltsev, 2002):

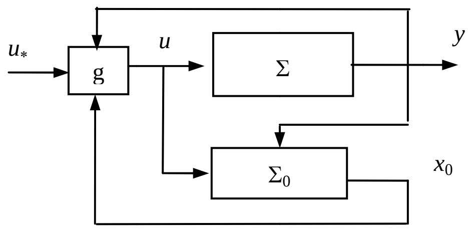

Fig. 1. Scheme for system Σ control

$$\Phi F = F_0\Phi + J_0 H, \qquad G_0 = \Phi G. \tag{6}$$

The system (3) is independent of the unknown vector $\vartheta(t)$, if the equality $\Phi L = 0$ holds. By analogy with (5), represent a bilinear term in (3) in the form $\sum_{i=1}^{m} u_i \sum_{j=1}^{n} G_0^j F_0^{ij} x_0$ for some matrices G_0^j and F_0^{ij}, $i = 1,2,...,m,\ j = 1,2,...,n$, which must be determined. It follows immediately from definition of the matrix Φ and (6) that the following relationships hold: $\Phi G^j = G_0^j$, $F^{ij} = F_0^{ij}\Phi$, $i = 1,2,...,m,\quad j = 1,2,...,n$.

3. SYSTEM Σ_0 DESIGN

The matrix Φ can be obtained as follows. Introduce the matrix L^0 of maximal row rank such that $L^0 L = 0$. The condition $\Phi L = 0$ implies the equality $\Phi = NL^0$ for some matrix N. Replace the matrix Φ in the first equation in (6) with NL^0 that gives $NL^0 F = F_0 NL^0 + J_0 H$ and transform it:

$$(N \quad -F_0 N \quad -J_0)\cdot((L^0 F)^T \quad (L^0)^T \quad H^T)^T = 0. \tag{7}$$

Expression (7) can be considered as an algebraic equation for the matrices N, F_0, and J_0. A solution to this equation gives the linear part of the system (3) described by the equation $\dot{x}_0(t) = F_0 x_0(t) + G_0 u(t) + J_0 y(t)$.

It the third step of the design, it is necessary to transform the obtained linear system into the bilinear one. Obtain the matrices F_0^{ij} from the algebraic equations $F^{ij} = F_0^{ij}\Phi$, $i = 1,2,...,m,\ j = 1,2,...,n$, and let $G_0^j = \Phi G^j$. Denote $F_0^i = \sum_{j=1}^{n} G_0^j F_0^{ij}$, $i = 1,2,...,m$, that concludes system (3) design.

4. SYSTEM Σ_* DESIGN

Let the function $G_0 u(t) + \sum_{i=1}^{m} u_i(t) F_0^i x_0(t)$ contains m', $m' \le m$, components of the vector *u*. Without loss of generality assume that these components are the first m' ones: $u_1, u_2, \ldots, u_{m'}$. Suppose also that for all $x \in \mathbf{R}^n$ the equality

$$rank\left(\frac{\partial}{\partial(u_1, u_2, \ldots, u_{m'})}(G_0 u(t) + \sum_{i=1}^{m} u_i(t) F_0^i x_0(t))\right) = c$$

holds for some *c*. Consider the case $s = c$ when $x_*(t) = x_0(t)$; then models (3) and (4) give the equality

$$\begin{aligned} &F_* x_*(t) + G_* u_*(t) + \sum_{i=1}^{m} u_i(t) F_*^i x_*(t) = \\ &F_0 x_0(t) + G_0 u(t) + J_0 y(t) + \sum_{i=1}^{m} u_i(t) F_0^i x_0(t). \end{aligned} \tag{8}$$

It is a basis for obtaining equation (2). Since expression (2) does not contain the vector x_*, let $F_* = 0$ and $F_*^i = 0$, $i = 1,2,...,m$. Besides to simplify a process of the system Σ_* control, let $G_* = I_{c\times c}$ that result in the following model of this system: $\dot{x}_{*i}(t) = u_{*i}(t),\ 1 \le i \le c$.

Thus to control the system (4), it is necessary to determine the first c components of the vector $u_*(t)$. The rest ones may be chosen arbitrary, for example, $u_{*i}(t) = 0,\ \ c+1 \le i \le m$.

5. CONTROL LOW DESIGN

Consider two cases. If $c = m'$ and $s = c$, then equation (8), equalities $F_* = 0$, $F_*^i = 0$, $i = 1,2,...,m$, $G_* = I_{c\times c}$ imply

$$F_0 x_0(t) + G_0 u(t) + J_0 y(t) + \sum_{i=1}^{m} u_i(t) F_0^i x_0(t) = u_*(t). \tag{9}$$

Since $c = m'$ and $s = c$, (9) is solvable for $u_1, u_2, \ldots, u_c$ and due to (2) these variables can be represented as follows:

$$u_i(t) = g_i(y(t), x_0(t), u_{*1}(t), \ldots, u_{*c}(t)),\ \ 1 \le i \le c = m', \tag{10}$$

for some functions g_1, $g_2, \ldots,$ g_c. Let $u_i(t) = u_{*i}(t)$, $m'+1 \le i \le m$, for the rest $m - m'$ components. In particular, if $m' = m = c$, $rank(G_0) = c$ and $G_0^{-1} F_0^i = 0$, $i = 1,2,...,m$, equality (10) takes a form $u(t) = G_0^{-1}(u_*(t) - F_0 x_0(t) - J_0 y(t))$.

If $c < m'$ and $s = c$, the right hand side of (8) contains redundant components of the vector $u(t)$. In this case one can obtain only some combinations of the vector $u(t)$ components of the form $\psi_i(u(t)) = g_i(y(t), x_0(t),\ u_{*1}(t), \ldots, u_{*c}(t))$, $1 \le i \le c$, for some functions ψ_1, $\psi_2, \ldots,$ ψ_c. In particular, when $G_0 = (G_* \ G_*')$, $rank(G_0) = rank(G_*) = c$, $G_0^{-L} F_0^i = 0$, $i = 1,2,...,m$, one obtains

$$(I_{c\times c} \ \ G_*^{-L} G_*')u(t) = G_*^{-L}(u_*(t) - F_0 x_0(t) - J_0 y(t))$$

where the matrix G_*^{-L} is left inverse to G_*.

6. CONCLUSION

In the framework of the fault accommodation problem in the systems described by model (1), the method of finding the control guaranteeing full decoupling with respect to faults effects is suggested. The feature of the method is that it uses linear algebraic tools only. This allows using simple mathematical packages to perform necessary calculations.

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