

## ADAPTIVE CONTROL OF CONDUCTIVITY INSIDE CSTR

VOJTESEK, J[iri]

**Abstract:** This paper deals with the Adaptive LQ control of the conductivity inside the real model of Continuous Stirred Tank Reactor (CSTR). The adaptive approach here is based on the choice of the External Linear Model (ELM) of originally nonlinear system, parameters of which are identified recursively during the control and parameters of the controller are recomputed too with the use of polynomial control synthesis together with Linear Quadratic (LQ) tracking which satisfies basic control requirements.

**Key words:** Adaptive Control, Recursive Identification, LQ approach, Polynomial Synthesis

### 1. INTRODUCTION

The experiments on the model as a small representation of the real system and simulation experiments are two basic techniques used for training and producing of controllers. Simulation on the computer (mathematical) model has big advantage in lower costs and time demands. On the other hand, some assumptions and simplifications must be introduced in order to reduce the complexity of the pure mathematical description of the system (Luyben, 1989). Real models are small and cheap representatives of the real, usually big and expensive, plants. Results are than in smaller scale but much closer to real ones. Disadvantage of the real model is that it is not as configurable as the simulation model in the computer.

Adaptive control (Astrom & Wittenmark, 1994) is well-known method with great theoretical background. In our case, an originally nonlinear system is replaced by the linearized description parameters of which are estimated recursively during the control. The resulted controller has in our case structure of the PID controller but its parameters are recomputed during the control according the identified ones.

### 2. REAL MODEL OF CSTR

The system under the consideration is Continuous Stirred Tank Reactor (CSTR) which is one part of the Multifunctional process control teaching system PCT40 from Armfield. This system includes several models and it is good teaching tool especially for process control teaching exercises. The schematic representation of the whole system is displayed in Figure 1.

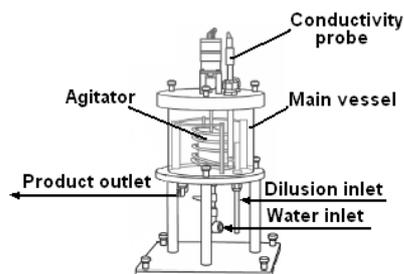


Fig. 1. Schematic representation of the Continuous Stirred Tank Reactor (CSTR)

The chemical process inside is dilution of 5% solution of the Sodium Chloride (NaCl) in the clear water. The content of the NaCl in the water affects the conductivity of the product which is measured by the conductivity probe. The system has

two input variables – volumetric flow rates of the dilution and clean water via two separate peristaltic pumps. Although both input variables could be used for the control, the volumetric flow rate of the dilution was set as constant and the volumetric flow rate of the clean water was used as an action value. The goal of the adaptive controller is to reduce the level of the conductivity inside via the volumetric flow rate of the clean water.

### 3. DYNAMIC ANALYSIS

Practically, the dynamic analysis for different step changes of the input variable was done at first and the achieved values of the conductivity for each step change in the stable time horizon was read as steady-state values for stable input step change. The six step changes of the input flow rate of the clean water were done – 30%, 45%, 60%, 75%, 90% and 100%.

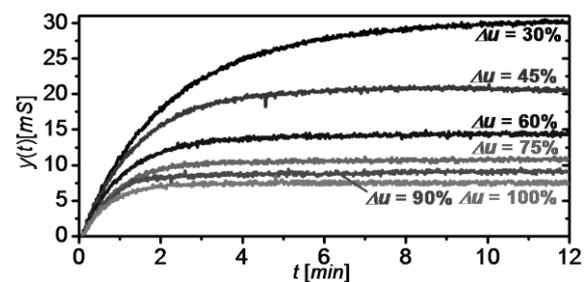


Fig. 2. Results of the dynamic analysis

As it can be seen from the steady-state analysis, the system has nonlinear behaviour mainly for high values of the input  $u$ . On the other hand, dynamic behaviour shows that the course of the output variable could be replaced by the first order transfer function. Nevertheless, due to some hidden behaviour which could not be readable in this working point, the second order transfer function with relative order one

### 4. ADAPTIVE CONTROL

The adaptive approach in this work is based on choosing an external linear model (ELM) of the original nonlinear system whose parameters are recursively identified during the control. Parameters of the resulted continuous controller are recomputed in every step from the estimated parameters of the ELM (Bobal et al., 2005).

The ELM used in this work is based on delta models (Mukhopadhyay et al., 1992) which is a special type of discrete-time models where each discrete difference is related to the sampling period  $T_s$ . It was proofed in (Stericker and Sinha 1993) that parameters of d-model approaches to the continuous ones.

Even though dynamic responses shown in can be described by first order transfer functions, the external linear model of this process in the continuous time, which is used in the control analysis, is of the second order with relative order one, i.e.

$$G(s) = \frac{b(s)}{a(s)} = \frac{Y(s)}{U(s)} = \frac{b_1s + b_0}{s^2 + a_1s + a_0} \quad (1)$$

where  $U(s)$  and  $Y(s)$  are Laplace transforms of the input and output variables.

Equation (1) has form of the differential equation for the identification:

$$y_{\delta}(k) = -a_1 y_{\delta}(k-1) - a_0 y_{\delta}(k-2) + b_1 u_{\delta}(k-1) + b_0 u_{\delta}(k-2) \quad (2)$$

where  $y_{\delta}$  is recomputed output to the  $\delta$ -model related to the sampling period  $T_v$ . The data vector for identification is then

$$\phi^T(k-1) = [-y_{\delta}(k-1), -y_{\delta}(k-2), u_{\delta}(k-1), u_{\delta}(k-2)] \quad (3)$$

and the vector of estimated parameters

$$\hat{\theta}^T(k) = [a'_1, a'_0, b'_1, b'_0] \quad (4)$$

could be computed from the ARX (Auto-Regressive eXogenous) model

$$y_{\delta}(k) = \hat{\theta}^T(k) \phi(k-1) \quad (5)$$

The Recursive Least-Squares (RLS) method is used for the parameter estimation in this work. The RLS method is well-known and widely used for the parameter estimation.

The control configuration with one degree-of-freedom (1DOF) was used for control synthesis – see Fig. 3.

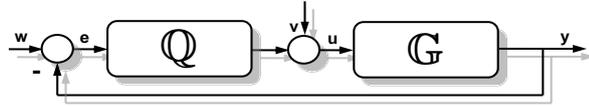


Fig. 3. 1DOF control configuration

$G$  denotes transfer function (1) of controlled plant,  $w$  is the reference signal (wanted value),  $v$  is disturbance,  $e$  is used for control error,  $u$  is control variable and  $y$  is a controlled output.

The transfer function of the controller has general form:

$$Q(s) = \frac{q(s)}{s \cdot \tilde{p}(s)} \quad (6)$$

where parameters of the polynomials  $\tilde{p}(s)$  and  $q(s)$  are computed from a Diophantine equation (Kucera 1993):

$$a(s) \cdot s \cdot \tilde{p}(s) + b(s) \cdot q(s) = d(s) \quad (7)$$

Polynomials  $a(s)$  and  $b(s)$  are known from the recursive identification and the polynomial  $d(s)$  on the right side of (7) is an optional stable polynomial. Roots of this polynomial are called poles of the closed-loop and their position affects quality of the control.

The polynomial  $d(s)$  is designed with the use of the Linear Quadratic (LQ) approach which is based on the minimization of the cost function

$$J_{LQ} = \int_0^{\infty} \{ \mu_{LQ} \cdot e^2(t) + \varphi_{LQ} \cdot \dot{u}^2(t) \} dt \quad (8)$$

where  $\varphi_{LQ} > 0$  and  $\mu_{LQ} \geq 0$  are weighting coefficients,  $e(t)$  is control error and  $\dot{u}(t)$  denotes difference of the input variable.

Polynomial  $d(s)$  in this case is  $d(s) = g(s) \cdot n(s)$  and polynomials  $n(s)$  and  $g(s)$  are computed from the spectral factorization

$$(a \cdot f)^* \cdot \varphi_{LQ} \cdot a \cdot f + b^* \cdot \mu_{LQ} \cdot b = g^* \cdot g \quad (9)$$

$$n^* \cdot n = a^* \cdot a$$

for control variable  $u(t)$  and disturbance  $v(t)$  from the ring of step functions  $f(s) = s$ . The transfer function of the controller in (6) is for this case

$$\tilde{Q}(s) = \frac{q_2 s^2 + q_1 s + q_0}{s \cdot (s^2 + p_1 s + p_0)} \quad (10)$$

and the polynomial  $d(s)$  is from (7) of the fifth degree.

## 5. CONTROL RESULTS

Proposed controller was tested by experiments on the real model which took 700 s and three changes of reference signal were done during this time. The adaptive controller could be

tuned via the choice of the weighting factor  $\phi_{LQ}$ , the second weighting factor  $\mu_{LQ}$  was constant during the control,  $\mu_{LQ} = 1$ . Two control studies for  $\phi_{LQ} = 0.005$  and 0.01 were done and results are shown in Fig. 4 and Fig. 5.

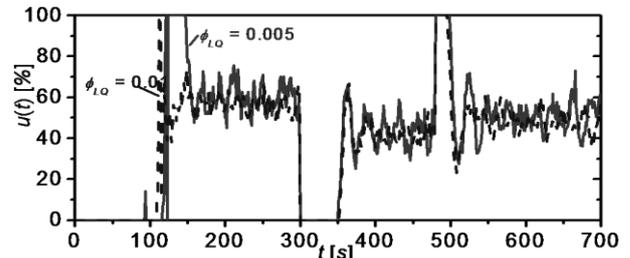


Fig. 4. The course of the input variable  $u(t)$  for various  $\phi_{LQ}$

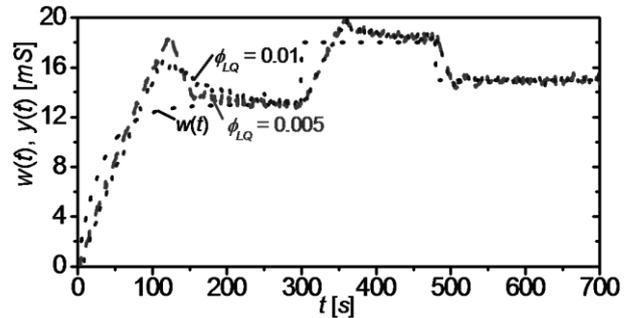


Fig. 5. The course of the input variable  $y(t)$  and reference signal  $w(t)$  for various  $\phi_{LQ}$

## 6. CONCLUSION

The paper presents application of adaptive controller to the real chemical process represented by the dilution of the salt in the clean water inside the continuous stirred-tank reactor as a part of multifunctional process control teaching system PCT40 and PCT41. The controller uses LQ approach and it could be tuned by the choice of the weighting factor  $\phi_{LQ}$ . Increasing value of this parameter results in lower overshoot at the beginning and the course of the action (input) variable which is more considerate to the peristaltic pump for the controller with  $\phi_{LQ} = 0.01$ . The problem at the beginning of the control is caused by inappropriate identification results because of the lack of information about the system. On the other hand, the controller produces stable and relatively quick response after this initial time. The main result of this work can be found in usability of the theoretically well described LQ adaptive control to the real system.

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