Annals of DAAAM for 2010 & Proceedings of the 21st International DAAAM Symposium, Volume 21, No. 1, ISSN 1726-9679
ISBN 978-3-901509-73-5, Editor B. Katalinic, Published by DAAAM International, Vienna, Austria, EU, 2010
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Annals of DAAAM International

ABOUT THE STABILITY OF THE MOTION OF A DYNAMIC SYSTEM IN A PARTICULAR CASE

DRAGOMIRESCU, C[ristian] G[eorge] & ILIESCU, V[ictor]

Abstract: *The paper is analysing a mechanical system, using the theory of the dynamical systems. Through the values of the characteristics of the system we point out the cases when the deterministic chaotically motions may occur. Using three different investigation methods the degree of the prediction accuracy is increased.*
Key words: *Chaos, Stability, Oscillator, Melnikov, Liapunov*

1. INTRODUCTION

Sometimes, motions described by precise laws with a reduced number of parameters are showing, at first sight, a random aspect of the motion. Highlighting the chaotically motions produced in deterministic conditions represented a conceptual revolution with applications in various domains: physics, chemistry, biology, geography, economics etc. The fact was also proven in day to day engineering.

The scientific works (Argyris et al., 1994), (Voinea & Stroe, 2000) are showing some specific characteristics of these motions: high sensitivity to initial conditions, non/repeatable motions, nonlinear differential equations with at least three independent variables, the fractal attractor in a finite domain.

Usually, the study of a deterministic chaotically motion is done using several methods: time history, time series, phase space portrait, Poincaré stroboscopic method, entropy, Liapunov exponents, power spectra, Melnikov method etc.

In the paper a mechanical system is analysed using three of these methods. In the process, the methods are developed with original contributions.

2. THE STUDY MODEL AND METHODS

The mechanical system used is shown in Fig. 1 and consists of a reversed damped pendulum attached to an oscillating frame. The differential motion equation is:

$$J_O\ddot{\theta} + c\dot{\theta} + k\theta - mag\sin\theta = a\xi_0\Omega^2\cos(\Omega t), \qquad (1)$$

where J_O is the mechanical inertial momentum, m – the mass of the body, c – the damper coefficient, k – the elasticity coefficient of the elastic element, g – the gravitational acceleration, ξ_0 – the amplitude of the harmonic oscillation, Ω - the beat of the perturbation.

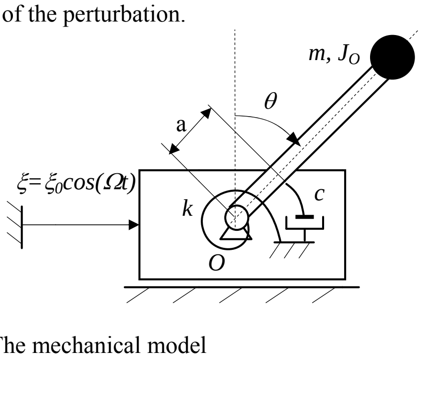

Fig. 1. The mechanical model

Using the normal approximations for small amplitude oscillations $\left(\cos\theta \approx 1, \sin\theta \approx 1 - \frac{\theta^3}{6}\right)$ and dividing the differential equation (1) by J_O, we obtain:

$$\ddot{\theta} + \alpha\dot{\theta} + \beta\theta + \gamma\theta^3 = \tilde{\xi}_0\cos(\Omega t), \qquad (2)$$

where $\alpha = \frac{c}{J_O}; \beta = \frac{k - mag}{J_O}; \gamma = \frac{mag}{6J_O}; \tilde{\xi}_0 = \frac{ma\xi_0\Omega^2}{J_O}$.

Due to the fact that the necessary and sufficient conditions for the occurrence of the chaotically motions are not yet determined, the paper is going to use for the study three of the most used criteria.

a) Phase space portrait is showing a closed curve (a loop) in the case of a periodic motion. If the motion is driven towards an equilibrium position or a periodic motion, the phase space portrait is going towards a critical point or a limit cycle. For a chaotically motion, the phase space portrait is becoming more complicated than in the previous two cases and the phenomena are no longer predictable.

b) Melnikov method is used in case of periodic motions and is analysing the conditions necessary in order for the stable and non-stable varieties of the same hyperbolic point to transversally intersect each odder. The usually used equation is:

$$\dot{x} = f(x) + \varepsilon \cdot g(x,t), \qquad (3)$$

where *f(x)* is a Hamiltonian field $\left(f_1 = \frac{\partial H}{\partial V}, f_2 = \frac{\partial H}{\partial V}\right)$ defined on R^2 and $\varepsilon g(x,t)$ is a small perturbation. If we admit the idea of the separation of the varieties $W^s\left(p_\varepsilon^{t_0}\right)$ and $W^n\left(p_\varepsilon^{t_0}\right)$ (for $\varepsilon \neq 0$ but small enough in the p^{t_0} point) in the transversal section on the homicline, Melnikov defined the function wearing his name:

$$M(t_0) = \int_{-\infty}^{+\infty} f[q^0(t,t_0)]\wedge g[q^0(t,t_0,t)]dt, \qquad (4)$$

where all the variables and parameters are those shown in (Argyris et al., 1994).

If the functions f and g are having the form: $f = \{\theta_2 \quad \beta\theta_1 - \gamma\theta_1^3\}$, $g = \left\{0 \quad \varepsilon\left(\tilde{\xi}_0\cos(\Omega t) - \alpha\theta_2\right)\right\}$, the Melnikov function becomes (Argyris et al., 1994):

$$M(t_0) = \pi\Omega\sqrt{\frac{2}{\gamma}}\,\tilde{\xi}_0\frac{\sin(\Omega t_0)}{ch\left(\frac{\pi\Omega}{2\sqrt{\beta}}\right)} - \frac{4}{3}\frac{\alpha}{\gamma}\beta\sqrt{\beta}, \qquad (5)$$

If M=0 and $\sin(\Omega t_0)=1$, the Holmes-Melnikov-Grenze function is obtained (Argyris et al., 1994):

$$\tilde{\xi}_{0cr} = \frac{4}{3}\frac{\alpha\beta}{\pi\Omega}\sqrt{\frac{\beta}{2\gamma}}\,\mathrm{ch}\frac{\pi\Omega}{2\sqrt{\beta}}\,. \qquad (6)$$

Using this function, the homoclinean bifurcation may be highlighted. If $\tilde{\xi}_0 < \tilde{\xi}_{0cr}$, the stable and non-stable varieties of the hyperbolic point cannot have a transversal intersection and, as a general conclusion, the chaotically motions may not occur. If $\tilde{\xi}_0 > \tilde{\xi}_{0cr}$, the trajectories become very complicated, like Smale horseshoe (Voinea & Stroe, 2000), and the chaotically motions may occur.

c) Liapunov exponents are a measure of the divergence of the phase space trajectories used to characterise the stability of the motion.

The differential equation (2) is equivalent with a system consisting of three differential equations of the first order:

$$\dot{\theta}_1 = \theta_2; \dot{\theta}_2 = -\alpha\theta_2 - \beta\theta_1 - \gamma\theta_1^3 + \tilde{\xi}_0\cos\theta_3; \dot{\theta}_3 = \Omega\,. \qquad (7)$$

Out of this system we obtain three Liapunov exponents. If at least one of them is positive: $L_i>0$ (i = 1, 2, 3) and the sum of all exponents is negative: $L_1 + L_2 + L_3 < 0$, a strange attractor appears and the motion is chaotically (Voinea & Stroe, 2000). It is to be mentioned that we are highlighting this property even if these conditions are not considered in the great majority of the scientific papers.

By numerical integration of the equation (2), in a defined time interval, an array of solutions $\{Z\}_i = \{\theta_1 \quad \theta_2 \quad \theta_3\}_i^T$ is obtained. If we consider for each interval (t_i, t_{i+1}) a small perturbation of the initial conditions $\{\delta\}_i = \{\delta\theta_1 \quad \delta\theta_2 \quad \delta\theta_3\}_i^T$ and we integrate again, for each time interval we obtain the array of the "perturbed solutions" $\{Z^*\}_i = \{\theta_1^* \quad \theta_2^* \quad \theta_3^*\}_i^T$.

In order to maximize the efficiency we propose that the perturbations to be proportional with the values of the functions at the t_i moment and several times smaller than the intermediate values of $\{Z\}_i$. We obtain the perturbations $\{\delta\}_i = \{\theta_{1i}\delta\theta_{10} \quad \theta_{2i}\delta\theta_{20} \quad \theta_{3i}\delta\theta_{30}\}_i^T$, where the initial perturbations (θ_{j0}, j =1, 2, 3) are about 10^{-2}.

Considering the above mentioned conditions, the Liapunov exponents are calculated as follows (Argyris et al., 1994):

$$L_N = \frac{1}{t_f}\sum_{i=1}^{N}\lg\frac{\left|\theta_{ji}^* - \theta_{ji}\right|}{\left|\theta_{ji}\delta\theta_{j0}\right|} \text{ (j = 1, 2, 3)}, \qquad (8)$$

where N is the number of time intervals.

Nr.	Ω	θ_{10}	θ_{20}	α	β	γ
1	0.8	0.3	0.8	0.15	-0.6	0.6
2	0.6	1.0	0.0	0.15	-0.6	0.6
3	0.6	0.0	0.0	0.15	-0.6	0.6
4	0.6	0.6	0.8	0.15	-0.6	0.6
Nr.	$\tilde{\xi}_0$	$\tilde{\xi}_{0cr}$	L_1	L_2	L_3	Fig.
1	0.15	0.08225	-2.0774	0.5844	0	2a
2	0.15	0.07574	-1.8180	1.4153	0	2b
3	0.15	0.07578	-1.7281	1.2986	0	2c
4	0.15	0.07576	-3.3082	-0.2479	0	2d

Tab. 1. The results of the study

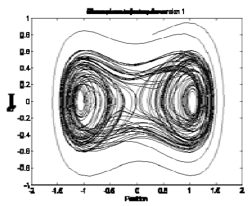

Fig. 2a-chaotic motion

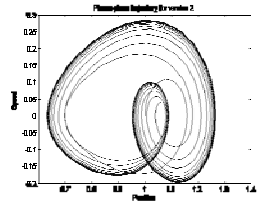

Fig. 2b-chaotic motion

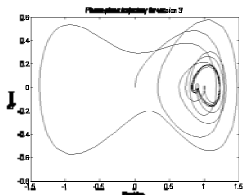

Fig. 2c-chaotic motion

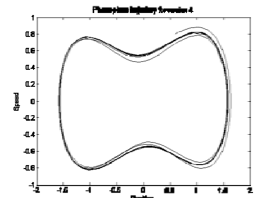

Fig. 2d-stable motion

Fig. 2. The phase space trajectories

In order to obtain the average values it is necessary to consider a large enough t_f.

The method may be called "approximate" due to the way of calculus. It is to be mentioned that in the scientific papers it is not presented in detail the calculus method for Liapunov exponents.

3. RESULTS AND CONCLUSIONS

In Tab. 1 we present the results obtained with the three methods used:

a) Phase space portrait – the trajectories in the phase space are crossing many times the stable limit cycle before the motion becomes stable;

b) Melnikov method – the information gathered with the Holmes-Melnikov-Grenze function (6) are not relevant, due to the fact that it is not reflecting the influence of the initial conditions (Dragomirescu & Iliescu, 2001);

c) Liapunov exponents – the motion is stable for $L_1<0$ and $L_2<0$ and chaotically for $L_1<0$ and $L_2>0$. The information obtained is accurate enough and consistent with scientific papers (Argyris et al., 1994), (Magnus et al., 2008). In all cases $L_3=0$, being consistent with the observations done by (Voinea & Stroe, 2000).

As an overall conclusion we consider that using all three methods above presented, in parallel, we may better qualitatively and quantitatively identify the domains where chaotically motions may occur. Otherwise it is difficult to accurately predict the evolution of the system, being well known that a system may have different behaviours: periodically motions, periodically windows, overlapping of motions having different natural frequencies etc.

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